Gravity, Horizons, and Open EFTs

引力、视界与有效场论开系

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Abstract

摘要

Wilsonian effective theories exploit hierarchies of scale to simplify the description of low-energy behaviour and play as central a role for gravity as for the rest of physics. They are useful both when hierarchies of scale are explicit in a gravitating system and more generally for understanding precisely what controls the size of quantum corrections in gravitational systems. But effective descriptions are also relevant for open systems (e.g. fluid mechanics as a long-distance description of statistical systems) for which the ”integrating out” of unobserved low-energy degrees of freedom complicate a straightforward application of Wilsonian methods. Observations performed only on one side of an apparent horizon provide examples where open-system descriptions also arise in gravitational physics. This chapter describes some early adaptations of Open Effective Theories (i.e. techniques for exploiting hierarchies of scale in open systems) in gravitational settings. Besides allowing the description of new types of phenomena (such as decoherence) these techniques also have an additional benefit: they sometimes can be used to resume perturbative expansions at late times and thereby to obtain controlled predictions in a regime where perturbative predictions otherwise generically fail.

威尔逊有效理论利用标度层级简化对低能行为的描述，其在引力领域和物理学其他领域中都占据核心地位。无论是当引力系统中存在明确的标度层级时，还是更一般地用来准确理解引力系统中量子修正的大小由什么决定，威尔逊有效理论都十分有用。但有效描述同样适用于开放系统 (例如统计力学中流体力学作为长程描述)，对这类系统，“积去”未被观测的低能自由度会给威尔逊方法的直接应用带来阻碍。在引力物理学中，仅在表观视界一侧进行观测就是开放系统描述的典型场景。本章介绍开放有效理论 (即开放系统中利用标度层级的方法) 在引力场景下的早期适配工作。这些方法除了能描述新型现象 (比如退相干) 之外，还有一项额外优势: 它们有时可用于重整 late time 微扰展开，从而在原本微扰预测通常失效的区域得到可控的预言。

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Open Systems and Gravity as a Medium

开放系统与作为介质的引力

Nature comes to us with a riot of scales: from the smallest known subatomic particles out to the observable universe as a whole. Because we do not yet know how Nature works at all scales a central question asks how badly unknown physics can interfere with predictions for physics we think we do understand. Wilsonian effective field theories (EFTs) are important because (at least so far) they provide the only known answer to this question.

自然界存在多种多样的尺度: 从已知最小的亚原子粒子，一直到整个可观测宇宙。由于我们目前还不了解自然界在所有尺度下的运行规律，一个核心问题是: 未知物理会对我们自认为已经理解的物理过程的预言产生多大影响。威尔逊有效场论 (EFT) 之所以重要，是因为 (至少到目前为止) 它是解答该问题的唯一已知方案。

They do so by quantifying how physics at a high energy M contributes to observables at a low energy E . A central result shows that most of the high-energy details only enter into low-energy observables suppressed by powers of the small ratio E/M and furthermore shows why these can be captured order-by-order in E/M - for all low-energy observables - as effective couplings within some form of a local Wilsonian action, or Hamiltonian

有效场论通过量化高能物理 M 如何贡献低能可观测量 E 实现这一点。一个核心结论表明，大部分高能细节仅会对低能可观测量产生贡献，且贡献被小比率 E/M 的幂次压低；此外该结论还说明，为何所有低能可观测量都可以按 E/M 的阶数逐阶处理——这些贡献可以被吸收为局域威尔逊作用量或哈密顿量中的有效耦合。

$$H_{\text{eff}} = \int \mathrm{d}^3 x \sum_s c_s \mathcal{O}_s(x) \,. \tag{1}$$

Here the effective couplings c_s are often proportional to $M_s^{4-d_s}$ where d_s is the scaling dimension of the local effective interaction operator $\mathcal{O}_s(x)$ that is built using only fields $\phi(x)$ that represent the low-energy states.

在此，有效耦合 c_s 通常正比于 $M_s^{4-d_s}$ ，其中 d_s 是局域有效相互作用算符 $\mathcal{O}_s(x)$ 的标度维度，该算符仅由描述低能态的场 $\phi(x)$ 构成。

In the normal treatment much discussion goes into how to compute the effective couplings c_s and interactions $\mathcal{O}_s(x)$ that are generated by particular types of known high-energy physics, taking for granted the existence of an appropriate H_{eff}. This is usually reasonable to do because the rules of quantum mechanics guarantee some sort of a Hamiltonian must exist, basically because energy conservation ensures that states that start at low energy remain at low energy (and so their evolution must be describable using the low-energy basis of field operators ϕ). The uncertainty principle then ensures that the low-energy effects of virtual high-energy states are local in time - at least order-by-order in E/M - from which (at least for relativistic systems) locality in space usually also follows. (See e.g. [1] for an extensive review.)

在常规处理中，大量讨论都聚焦于如何计算由特定已知高能物理产生的有效耦合 c_s 和相互作用 $\mathcal{O}_s(x)$，并默认合适的 H_{eff} 必然存在。这种处理通常是合理的，因为量子力学的规则保证某种形式的哈密顿量一定存在: 根本原因是能量守恒保证初始处于低能的态会始终保持低能，因此它们的演化一定可以用低能场算符基矢 ϕ 描述。不确定原理进一步保证虚高能态的低能效应在时间上是局域的——至少按 E/M 逐阶处理时成立——由此 (至少对相对论系统而言) 空间上的局域性通常也能得到保证。(相关综述参见例如文献 [1]。)

The theory of Open Quantum Systems (see for example [2]) provides the general framework for describing situations where only a subset of states are observed within a wider quantum system, and because of this one might expect EFTs to closely resemble open systems. This turns out not to be true. In particular it is in general not true that an effective Hamiltonian can capture how unobserved degrees of freedom influence others that we do measure.

开放量子系统理论 (例如参见文献 [2]) 为更大量子系统中仅观测部分子态的情形提供了通用框架，因此人们可能会期望有效场论和开放系统十分相似。但事实并非如此。尤其是，有效哈密顿量一般并不足以描述未被观测的自由度对我们可测自由度的影响。

EFTs turn out to be a very special case where measured and unmeasured degrees of freedom are distinguishable from one another by the eigenvalues of a conserved charge (the value of their energy, plus possibly other conserved quantities), and this plays a crucial role when inferring the existence of an effective Hamiltonian. For general open systems the measured and unmeasured degrees of freedom can both have similar energies and can exchange information and mutually entangle, making the evolution of any observed subsector usually more complicated. This difference is why open systems can exhibit rich phenomena like thermalization and decoherence not normally seen in a simple Wilsonian low-energy/high-energy split.

实际上有效场论是一个非常特殊的情形: 其中可测和不可测自由度可以通过守恒荷的本征值 (能量值，外加可能的其他守恒量) 彼此区分，这一点在推导出有效哈密顿量存在的过程中发挥了关键作用。对于一般开放系统，可测和不可测自由度可以具有相近的能量，还可以交换信息、相互纠缠，这使得任何可观测子区域的演化通常都更为复杂。这种差异正是开放系统可以展现热化、退相干等丰富现象的原因，这些现象在简单的威尔逊低能/高能拆分中通常不会出现。

For quantum gravity the relative richness of open systems compared to Wilsonian evolution is not just an academic point. This is because the existence of horizons can famously prevent low-energy degrees of freedom from interacting (and being measured) by other low-energy degrees of freedom. The description appropriate for such situations is necessarily open and this is partly why horizons can have surprising quantum implications.

对于量子引力而言，开放系统相比威尔逊演化更具丰富性这一点并不仅仅是学术问题。众所周知，这是因为视界的存在会阻碍低能自由度之间的相互作用 (以及测量)，适合这类情形的描述必然是开放的，这也是视界能够产生惊人量子效应的部分原因。

Because near-horizon systems are open, their effective description can differ from standard Wilsonian reasoning and it is worth exploring what this can mean. This involves exploring how predictions are made for open systems and identifying the differences from the standard Wilsonian approach. Since some steps differ from standard Wilsonian practice, special attention must be paid to the domain of validity for the approximations that are used. Of particular interest are features of Wilsonian intuition (such as locality) that might not generalize to open systems more broadly.

由于近视界系统是开放的，其有效描述可能与标准的威尔逊推理不同，因此值得探讨这一点的意义。这需要研究开放系统如何做出预言，并明确它和标准威尔逊方法的区别。由于部分步骤和标准威尔逊实践不同，必须特别关注所用近似的适用范围。尤其值得关注的是，威尔逊直觉中的部分特征 (例如局域性) 可能无法推广到更一般的开放系统中。

This chapter summarizes some of those issues but also describes how the systematic study of open systems brings a welcome bonus that is interesting in its own right: it provides a method for dealing with 'secular growth' problems often faced by quantum systems in gravitational fields. Secular growth is the gravitational version of something that is also generic elsewhere in physics [1] perturbative predictions break down at sufficiently late times because there is always a time for which

本章总结了上述部分问题，同时也说明了对开放系统的系统研究还能带来一个本身就十分有趣的额外收获：它提供了一种解决引力场中量子系统常遇到的“长期增长”问题的方法。长期增长是物理学其他领域也普遍存在的现象在引力中的版本 [1]：微扰预言在足够晚的时间会失效，因为总会存在某个时间使得

$$U(t) = e^{-i(H_0+H_{\rm int})t} \simeq e^{-iH_0 t}\left(1 - iH_{\rm int}\, t\right) \tag{2}$$

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[1] Scattering of wave packets of particles is one of the few physical questions where late-time interactions need not be a problem because packet separation itself turns off late-time interactions.

[1] 粒子波包散射是少数几个晚期相互作用不会成为问题的物理问题之一，因为波包分离本身就会关闭晚期相互作用。

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no matter how small $H_{\rm int}$ is compared with H_0. As we review below, one of the reasons for using open-system tools is they can sometimes allow reliable late-time predictions to be made even when $H_{\rm int}\, t$ is order unity, even if the only available calculations rely on perturbing in $H_{\rm int}$.

无论 $H_{\rm int}$ 相比于 H_0 有多小。正如我们下文回顾的，使用开系统方法的原因之一是，即便唯一可用的计算依赖于对 $H_{\rm int}$ 的微扰，当 $H_{\rm int}\, t$ 为一阶量级时，这些方法有时仍能给出可靠的晚期预测。

Such tools are particularly important when drawing late-time inferences about quantum systems in gravitational fields because intuition about these systems is often based on simple examples that interact only with the background gravitational field and do not self-interact. It is tempting to think that this intuition should remain reliable in the presence of self-interactions because these self-interactions might be chosen as small as one wants. But smaller couplings only really delay when perturbation theory fails rather than remove its failure altogether. So more robust means for making prediction are required when late-time behaviour is important. Quantum-gravitational problems for which late-time evolution is crucial include black hole information loss and some issues of eternal inflation.

> 这类工具对于推导引力场中量子系统的晚期推论尤为重要，因为我们对这类系统的直觉往往来自简单示例：这些示例仅与背景引力场相互作用，本身不存在自相互作用。人们很容易会认为，既然自相互作用可以任意取小，这套直觉在存在自相互作用时依然可靠。但更小的耦合实际上只是推迟了微扰论失效的时间，而非彻底消除它的失效。因此当需要研究晚期行为时，我们需要更稳健的预测方法。量子引力问题中，黑洞信息丢失和永恒暴涨的若干问题都属于晚期演化至关重要的情况。

This chapter summarizes some early steps that have been taken applying open-system techniques to gravity (for other preliminary applications in particle physics and gravity respectively, see [3] and [4]), usually with late-time evolution specifically in mind. We do so from a purely personal perspective with the goal of laying out the conceptual framework as clearly as we can, rather than trying to survey exhaustively everything that has been done. We inevitably will have missed describing some gems within the literature along the way and apologize in advance - both to the authors and to you, the reader - for doing so.

> 本章总结了将开系统技术应用于引力的一些早期探索 (粒子物理和引力领域的其他初步应用分别参见文献 [3] 和 [4])，这些应用通常都专门聚焦于晚期演化。我们此处完全从个人视角出发，目标是尽可能清晰地梳理概念框架，而非穷尽调研已完成的所有工作。行文过程中我们不可避免会遗漏文献中的不少精彩工作，在此提前向相关作者与读者您致歉。

Open Quantum Systems

开放量子系统

The generic open-system problem restricts attention to a subset of observables lying within some sector A within the full system's Hilbert space. The goal is to compute how these observables evolve in time, including in particular how this is influenced by their interactions with all of the other unmeasured degrees of freedom.

> 一般开放系统问题仅关注整个系统希尔伯特空间中位于某分域 A 内的可观测量子集。我们的目标是计算这些可观测量如何随时间演化，尤其是它们与所有其他未被测量的自由度之间的相互作用对演化产生的影响。

For the reasons alluded to above, one usually does not seek to construct a universal H_{eff} that captures the influence of all unmeasured degrees of freedom. One instead sets up and solves a master equation that describes the time evolution of the state of the measured subsystem and uses this to predict how measurements evolve.

基于上文提到的原因，我们通常不需要构造一个能描述所有未测量自由度影响的普适 $H_{\rm eff}$ ，而是建立并求解一个主方程，它描述被测子系统状态的时间演化，并用它来预测测量结果的演化规律。

To see how this works, suppose the Hilbert space for the quantum system of interest contains two sectors, A and B , and measurements are only performed in sector A (the ' system') while sector B (the ' environment') remains unmeasured. For simplicity consider the case where the full system's Hilbert space can be written as a product of states in sector A and those in B , so a basis of states in the full system can be decomposed as

为说明这套方法的原理，假设我们感兴趣的量子系统的希尔伯特空间包含两个分域 A 和 B ，仅在分域 A (即“系统”) 中进行测量，而分域 B (即“环境”) 始终不被测量。为简化讨论，我们考虑全系统希尔伯特空间可写为分域 A 和分域 B 的态空间直积的情况，因此全系统的态基可以分解为

$$|a, b\rangle =| a\rangle \otimes |b\rangle. \tag{3}$$

Observables involving only sector A are hermitian operators that can be written

仅涉及分域 A 的可观测量都是可写为如下形式的厄米算符

$$\mathcal{O}_A = O_A \otimes I_B \;\; \text{with matrix elements } \langle a, b\,|\mathcal{O}_A|\, a', b'\rangle = \langle a\,|O_A|\, a'\rangle\, \delta_{bb'}. \tag{4}$$

The focus in what follows is on describing the evolution of the system's state, as described by its density matrix

接下来我们将重点描述系统状态的演化，由密度矩阵刻画

$$\hat{\rho} \; := \; \sum_I p_I\, |I\rangle\langle I| \; := \; \sum_{ab} p_{ab}\, |a, b\rangle\langle a, b|\,, \tag{5}$$

where $p_I = p_{ab}$ is the probability to find the system in state $|I\rangle =| a, b\rangle$. As usual, the sum over mutually exclusive probabilities must give one so $\mathrm{Tr}\,\hat{\rho} = \sum_I p_I = 1$. A ' pure' state corresponds to the special case where the system' s state $|\psi\rangle$ is precisely known and so

其中 $p_I = p_{ab}$ 是系统处于态 $|I\rangle =| a, b\rangle$ 的概率。按惯例，所有互斥态的概率之和必须为 1，即 $\mathrm{Tr}\,\hat{\rho} = \sum_I p_I = 1$。“纯态”对应系统状态 $|\psi\rangle$ 完全已知的特殊情况，因此满足

$$\hat{\rho} = |\psi\rangle\langle\psi| \quad (\text{pure state})\,. \tag{6}$$

Condition (6) being true for some (normalized) state $|\psi\rangle$ is equivalent to the condition $\hat{\rho}^2 = \hat{\rho}$ and because all probabilities satisfy $0 \le p_{ab} \le 1$ this is also equivalent to the condition $\mathrm{Tr}\,\hat{\rho}^2 = \mathrm{Tr}\,\hat{\rho} = 1$. When $\hat{\rho}^2 \neq \hat{\rho}$ the state is said to be ' mixed' and there is no $|\psi\rangle$ for which (6) is true.

存在某个 (归一化) 态 $|\psi\rangle$ 满足条件 (6) 等价于条件 $\hat{\rho}^2 = \hat{\rho}$ ，又因为所有概率满足 $0 \le p_{ab} \le 1$ ，因此也等价于条件 $\mathrm{Tr}\,\hat{\rho}^2 = \mathrm{Tr}\,\hat{\rho} = 1$ 。当 $\hat{\rho}^2 \neq \hat{\rho}$ 时，该状态被称为“混合态”，不存在满足 (6) 的 $|\psi\rangle$ 。

Expectation values for observables are computed using $\hat{\rho}$ by evaluating [2]

可观测量的期望值可利用 $\hat{\rho}$ 通过计算 [2] 得到

$$\langle \hat{\mathcal{O}} \rangle = \sum_I p_I \langle I \,|\hat{\mathcal{O}}|\, I \rangle = \mathrm{Tr}\left(\hat{\rho}\hat{\mathcal{O}}\right). \tag{7}$$

Within the Schrödinger picture this acquires a time-dependence through the time-dependence of $\hat{\rho}(t)$, which is described by the evolution equation

在薛定谔绘景下，由于 $\hat{\rho}(t)$ 含时，密度矩阵也随之含时，其演化由下述演化方程描述

$$i\partial_t \hat{\rho} = [H, \hat{\rho}], \tag{8}$$

where H is the system’s Hamiltonian. Equation (8) is equivalent to the usual Schrödinger evolution $i\partial_t |\psi(t)\rangle = H|\psi(t)\rangle$ for a pure state satisfying (6).

其中 H 是系统的哈密顿量。方程 (8) 等价于满足 (6) 的纯态的常规薛定谔演化 $i\partial_t |\psi(t)\rangle = H|\psi(t)\rangle$ 。

Assuming the initial condition $\hat{\rho}(t = t_0) = \hat{\rho}_0$, Eq. (8) is formally solved by

给定初始条件 $\hat{\rho}(t = t_0) = \hat{\rho}_0$ ，方程 (8) 的形式解为

$$\hat{\rho}(t) = \hat{U}(t, t_0)\,\hat{\rho}_0 \hat{U}^*(t, t_0), \tag{9}$$

where Hermiticity of H ensures $\hat{U}(t, t_0) = \exp[-iH(t - t_0)]$ is unitary and so satisfies $\hat{U}(t, t_0)\,\hat{U}^*(t, t_0) = 1$. Furthermore $\hat{U}(t, t_1)\,\hat{U}(t_1, t_0) = \hat{U}(t, t_0)$ while

其中 H 的厄米性保证 $\hat{U}(t, t_0) = \exp[-iH(t - t_0)]$ 是幺正算符，因此满足 $\hat{U}(t, t_0)\,\hat{U}^*(t, t_0) = 1$ 。此外，$\hat{U}(t, t_1)\,\hat{U}(t_1, t_0) = \hat{U}(t, t_0)$ 且

$$i\partial_t \hat{U}(t, t_0) = H\hat{U}(t, t_0) \quad \text{with initial condition } \hat{U}(t_0, t_0) = I. \tag{10}$$

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[2] The ’hat’ symbol is meant to emphasize that the operator is written in Schrödinger picture.

[2] 我们用 hat 符号强调该算符是写在薛定谔绘景下的。

So far this is all bog-standard quantum mechanics. The ’open-system’ part of the story starts once we assume the only observables of interest have the form (4), expressing that they probe only sector A .

到目前为止都是标准量子力学的内容。当我们假设所有感兴趣的可观测量都仅探测分域 A ，即具有 (4) 的形式时，故事的“开放系统”部分才正式开始。

For open systems the goal is to understand how measurements restricted to sector A evolve in a way that refers as much as possible only to the measured sector A . A key tool to this end is the 'reduced' density matrix, defined by tracing the full density matrix over the unmeasured sector B :

对于开放系统，我们的目标是理解限制在 A 扇区的测量如何以尽可能仅涉及被测扇区 A 的方式演化。为此，一个核心工具是「约化」密度矩阵，它通过对全密度矩阵在未测量扇区 B 上求迹定义：

$$\hat{\rho}_A \; := \; \mathrm{tr}_B \hat{\rho} \quad \text{so} \quad \langle a \,|\hat{\rho}_A|\, a' \rangle = \sum_b \langle a, b \,|\hat{\rho}|\, a', b\rangle. \tag{11}$$

This is useful because it defines an operator that acts only in sector A whose time evolution completely determines the time-dependence of measurements of type (4) that also act only in sector A , as may be seen from expressions like

这一工具十分有用，因为它定义了一个仅作用于 A 扇区的算符，该算符的时间演化完全决定了同样仅作用于 A 扇区的 (4) 类测量的时间相关性，这可以从下式看出

$$\left\langle \hat{\mathcal{O}}_A \right\rangle \; := \; \mathrm{Tr}\left[\hat{\rho}\,(t)\,\hat{\mathcal{O}}_A\right] = \sum_{aa'} \sum_b \langle a, b\,|\hat{\rho}|\, a', b\rangle \left\langle a' \left|\hat{\mathcal{O}}_A\right| a \right\rangle = \mathrm{tr}_A \left[\hat{\rho}_A\,(t)\,\hat{\mathcal{O}}_A\right].$$

(12)

In principle the evolution of $\hat{\rho}_A$ is obtained by taking the trace of (8) over sector B and integrating the result. In practice this is not so useful because the right-hand side of the resulting equation depends on the entire density matrix $\hat{\rho}$ (including what the environmental sector is doing) rather than just being a function of $\hat{\rho}_A$. Happily, this is a problem that has been solved in some generality, as is possible because both the Liouville evolution (8) and the projection from $\hat{\rho}$ onto $\hat{\rho}_A$ are linear operations on the space of density matrices. One must set up and solve the evolution equation for the state of the environment and use this to eliminate the environment from the expression for $\partial_t \hat{\rho}$.

原则上，$\hat{\rho}_A$ 的演化可以通过对 (8) 式在 B 扇区求迹并积分结果得到。但实际中这种方法并不实用，因为所得方程的右侧依赖于整个密度矩阵 $\hat{\rho}$ (包含环境扇区的行为)，而非仅为 $\hat{\rho}_A$ 的函数。幸运的是，这个问题已经在一定普适性下得到解决，这之所以可行，是因为刘维尔演化 (8) 和从 $\hat{\rho}$ 到 $\hat{\rho}_A$ 的投影都是密度矩阵空间上的线性操作。我们需要建立并求解环境态的演化方程，再利用它将环境从 $\partial_t \hat{\rho}$ 的表达式中消去。

To this end define the super-operator $\mathcal{P}$ to act on a general operator $\mathcal{O}$ by

为此，定义超算符 $\mathcal{P}$ 对一般算符 $\mathcal{O}$ 的作用如下：

$$\mathcal{P}\,(\mathcal{O}) \; := \; \mathrm{tr}_B\,(\mathcal{O}) \otimes \rho_B. \tag{13}$$

Here ρ_B is a density matrix that characterizes the state of sector B (the environment) that is specified in more detail later. Because $\mathrm{tr}_B \rho_B = 1$ it follows that $\mathcal{P}$ is a projection operator: $\mathcal{P}^2 = \mathcal{P}$. It also satisfies $\mathcal{P}\,(\rho_A \otimes \rho_B) = \rho_A \otimes \rho_B$ and so $\mathcal{P}$ does nothing if it acts on a density matrix for which sectors A and B are uncorrelated. Finally, $\mathcal{P}\,[\hat{\rho}\,(t)] = \hat{\rho}_A\,(t) \otimes \rho_B$, where $\hat{\rho}_A\,(t) = \mathrm{tr}_B \hat{\rho}$ is the reduced density matrix whose time evolution we seek.

此处 ρ_B 是描述 B 扇区 (环境) 状态的密度矩阵，我们后续会对其做更详细的说明。由 $\mathrm{tr}_B\rho_B = 1$ 可得 $\mathcal{P}$ 是投影算符: $\mathcal{P}^2 = \mathcal{P}$ 。它还满足 $\mathcal{P}(\rho_A \otimes \rho_B) = \rho_A \otimes \rho_B$ ，因此当 $\mathcal{P}$ 作用在 A 扇区和 B 扇区无关联的密度矩阵上时，不会改变该密度矩阵。最后，$\mathcal{P}[\hat{\rho}(t)] = \hat{\rho}_A(t) \otimes \rho_B$ ，其中 $\hat{\rho}_A(t) = \mathrm{tr}_B\hat{\rho}$ 是我们要求其时间演化的约化密度矩阵。

Because $\mathcal{P}$ is a projection super-operator, the same is also true for its complement $\mathcal{Q} := 1 - \mathcal{P}$: i.e. $\mathcal{Q}$ satisfies $\mathcal{Q}^2 = \mathcal{Q}$ and $\mathcal{P}\mathcal{Q} = \mathcal{Q}\mathcal{P} = 0$. In this same language time evolution is also described by a linear super-operator, since (8) can be written $\partial_t\hat{\rho} = \mathcal{L}_t(\hat{\rho})$ where

由于 $\mathcal{P}$ 是投影超算符，它的补 $\mathcal{Q} := 1 - \mathcal{P}$ 同样也是投影超算符: 即 $\mathcal{Q}$ 满足 $\mathcal{Q}^2 = \mathcal{Q}$ 和 $\mathcal{P}\mathcal{Q} = \mathcal{Q}\mathcal{P} = 0$ 。在这套语言中，时间演化也可以用线性超算符描述，因为 (8) 式可以写为 $\partial_t\hat{\rho} = \mathcal{L}_t(\hat{\rho})$ ，其中

$$\mathcal{L}_t(\mathcal{O}) := -i[H, \mathcal{O}]. \tag{14}$$

The evolution of $\mathcal{P}(\hat{\rho})$ and $\mathcal{Q}(\hat{\rho})$ with time is easily found by projecting the evolution equation (8) using $\mathcal{P}$ and $\mathcal{Q}$. Using $\mathcal{P}(\partial_t\hat{\rho}) = \partial_t\mathcal{P}(\hat{\rho})$ and $\mathcal{P} + \mathcal{Q} = 1$ allows (8) to be rewritten as the pair of equations

$\mathcal{P}(\hat{\rho})$ 和 $\mathcal{Q}(\hat{\rho})$ 随时间的演化可以通过用 $\mathcal{P}$ 和 $\mathcal{Q}$ 投影演化方程 (8) 轻松得到。利用 $\mathcal{P}(\partial_t\hat{\rho}) = \partial_t\mathcal{P}(\hat{\rho})$ 和 $\mathcal{P} + \mathcal{Q} = 1$，(8) 式可以改写为下述方程组:

$$\partial_t\mathcal{P}(\hat{\rho}) = \mathcal{P}(\partial_t\hat{\rho}) = \mathcal{P}\mathcal{L}_t(\hat{\rho}) = \mathcal{P}\mathcal{L}_t\mathcal{P}(\hat{\rho}) + \mathcal{P}\mathcal{L}_t\mathcal{Q}(\hat{\rho}) \ (15) \tag{15}$$

$$\text{and } \partial_t\mathcal{Q}(\hat{\rho}) = \mathcal{Q}(\partial_t\hat{\rho}) = \mathcal{Q}\mathcal{L}_t(\hat{\rho}) = \mathcal{Q}\mathcal{L}_t\mathcal{P}(\hat{\rho}) + \mathcal{Q}\mathcal{L}_t\mathcal{Q}(\hat{\rho}) .$$

Now comes the main point: the second of these equations can be used to eliminate $\mathcal{Q}(\hat{\rho})$ from the right-hand side of the first equation, thereby obtaining an evolution equation that involves only $\mathcal{P}(\hat{\rho})$. This solves our problem of setting up an evolution equation for the reduced matrix $\hat{\rho}_A$ to the extent that we can use $\mathcal{P}(\hat{\rho})$ as a proxy for $\hat{\rho}_A$, and these indeed would agree at an initial time t_0 if the initial state were assumed to be uncorrelated:

现在要点来了: 可以用第二个方程消去第一个方程右手边的 $\mathcal{Q}(\hat{\rho})$ ，从而得到一个仅含 $\mathcal{P}(\hat{\rho})$ 的演化方程。只要我们能用 $\mathcal{P}(\hat{\rho})$ 约化矩阵 $\hat{\rho}_A$ 的替代，这就解决了为 $\hat{\rho}_A$ 建立演化方程的问题；若假设初态不存在关联，二者在初始时刻 t_0 确实相等:

$$\hat{\rho}(t_0) = \hat{\rho}_A(t_0) \otimes \rho_B. \tag{16}$$

In general the evolution of $\mathcal{P}(\hat{\rho}(t)) = \hat{\rho}_A(t) \otimes \rho_B$ does not agree with that of $\hat{\rho}(t)$ because for $\mathcal{P}(\hat{\rho})$ the environment (sector B) does not evolve. But $\hat{\rho}$ and $\mathcal{P}(\hat{\rho})$ nonetheless by construction agree with one another once sector B is traced out and so agree on time-dependence for observables acting only in A .

一般来说，$\mathcal{P}(\hat{\rho}(t)) = \hat{\rho}_A(t) \otimes \rho_B$ 的演化与 $\hat{\rho}(t)$ 并不一致，因为对 $\mathcal{P}(\hat{\rho})$ 而言，环境 (扇区 B) 不发生演化。但根据构造，对扇区 B 求迹后，$\hat{\rho}$ 与 $\mathcal{P}(\hat{\rho})$ 必然相等，因此二者在仅作用于 A 的可观测量的时间依赖上是一致的。

Because Eqs. (15) are linear they can be solved fairly explicitly. If we define the super-operator $\mathcal{G}(t,s)$ as the solution to $\partial_t \mathcal{G}(t,s) = \mathcal{Q}\mathcal{L}_t \mathcal{G}(t,s)$ with initial condition $\mathcal{G}(t,t) = 1$ then $\mathcal{G}(t,s)$ is given explicitly by

由于方程 (15) 是线性的，可以得到相当明确的解。如果我们将超算符 $\mathcal{G}(t,s)$ 定义为初始条件为 $\mathcal{G}(t,t) = 1$ 时 $\partial_t \mathcal{G}(t,s) = \mathcal{Q}\mathcal{L}_t \mathcal{G}(t,s)$ 的解，那么 $\mathcal{G}(t,s)$ 可以明确写为

$$
\begin{aligned}
\mathcal{G}(t,s) &= 1 + \sum_{n=1}^{\infty} \int_s^t \mathrm{d}s_1 \cdots \int_s^{s_{n-1}} \mathrm{d}s_n \mathcal{Q}\mathcal{L}_{s_1} \cdots \mathcal{Q}\mathcal{L}_{s_n} \\
&= 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int_s^t \mathrm{d}s_1 \cdots \int_s^t \mathrm{d}s_n \mathcal{T}\left[\mathcal{Q}\mathcal{L}_{s_1} \cdots \mathcal{Q}\mathcal{L}_{s_n}\right]
\end{aligned}
\tag{17}
$$

where $\mathcal{T}$ denotes time-ordering of the $\mathcal{Q}\mathcal{L}_{s_i}$. This allows the solution for $\mathcal{Q}[\hat{\rho}(t)]$ with initial condition $\mathcal{Q}[\hat{\rho}(t_0)] = \mathcal{Q}(\hat{\rho}_0)$ to be written

其中 $\mathcal{T}$ 表示 $\mathcal{Q}\mathcal{L}_{s_i}$ 的时间排序。由此，初始条件为 $\mathcal{Q}[\hat{\rho}(t_0)] = \mathcal{Q}(\hat{\rho}_0)$ 时 $\mathcal{Q}[\hat{\rho}(t)]$ 的解可以写作

$$
\mathcal{Q}[\hat{\rho}(t)] = \mathcal{G}(t,t_0)\mathcal{Q}(\hat{\rho}_0) + \int_{t_0}^t \mathrm{d}s \mathcal{G}(t,s)\mathcal{Q}\mathcal{L}_s \mathcal{P}[\hat{\rho}(s)],
\tag{18}
$$

as can be verified by explicit differentiation, using $\partial_t \mathcal{G}(t,s) = \mathcal{Q}\mathcal{L}_t \mathcal{G}(t,s)$. Inserting this into the first of Eqs. (15) gives the desired evolution equation for $\mathcal{P}(\hat{\rho})$

这可以通过利用 $\partial_t \mathcal{G}(t,s) = \mathcal{Q}\mathcal{L}_t \mathcal{G}(t,s)$ 求导直接验证，将其代入方程 (15) 的第一个式子，就得到了 $\mathcal{P}(\hat{\rho})$ 满足的期望演化方程

$$
\partial_t \mathcal{P}[\hat{\rho}(t)] = \mathcal{P}\mathcal{L}_t \mathcal{P}[\hat{\rho}(t)] + \mathcal{P}\mathcal{L}_t \mathcal{G}(t,t_0)\mathcal{Q}(\rho_0) + \int_{t_0}^t \mathrm{d}s \mathcal{K}(t,s)[\hat{\rho}(s)],
\tag{19}
$$

where $\mathcal{K}(t,s) = \mathcal{P}\mathcal{L}_t \mathcal{G}(t,s)\mathcal{Q}\mathcal{L}_s \mathcal{P}$. The second term on the right-hand side vanishes for the uncorrelated initial condition $\rho_0 = \hat{\rho}(t_0) = \hat{\rho}_A(t_0) \otimes \rho_B$ since this satisfies $\mathcal{P}(\rho_0) = \rho_0$ and so $\mathcal{Q}(\rho_0) = 0$.

其中 $\mathcal{K}(t,s) = \mathcal{P}\mathcal{L}_t \mathcal{G}(t,s)\mathcal{Q}\mathcal{L}_s \mathcal{P}$ 。对于无关联初始条件 $\rho_0 = \hat{\rho}(t_0) = \hat{\rho}_A(t_0) \otimes \rho_B$ ，右手边第二项等于零，因为该条件满足 $\mathcal{P}(\rho_0) = \rho_0$ ，因此 $\mathcal{Q}(\rho_0) = 0$ 。

Equation (19) is an integro-differential master equation - initially due to Nakajima and Zwanzig [5,6] - that contains the same information for sector A as does the original evolution equation (8). It is also typically no easier to solve. Its main virtue is that $\hat{\rho}(t)$ only appears in it through the combination $\mathcal{P}[\hat{\rho}(t)]$ and so it expresses $\partial_t \hat{\rho}_A$ directly in terms of $\hat{\rho}_A$ itself.

方程 (19) 是一个积分微分主方程——最初由中岛 (Nakajima) 和茨万齐格 (Zwanzig) 提出 [5,6]——它对于区间 A 包含的信息与原始演化方程 (8) 完全相同，通常求解难度也没有降低。它的主要优点是，$\hat{\rho}(t)$ 仅通过组合项 $\mathcal{P}[\hat{\rho}(t)]$ 出现在方程中，因此它直接用 $\hat{\rho}_A$ 本身表示了 $\partial_t \hat{\rho}_A$ 。

The final result is most useful when the interaction between sectors A and B can be treated perturbatively. To this end we write

当区间 A 和 B 之间的相互作用可以用微扰法处理时，最终结论最为有用。为此我们写出

$$H = H_0 + \widehat{V} \text{ with } H_0 = H_A + H_B \tag{20}$$

and switch to the interaction picture so $\mathcal{O}(t) = e^{iH_0(t-t_0)}\widehat{\mathcal{O}}e^{-iH_0(t-t_0)}$, so that state evolution is controlled only by $V(t) = e^{iH_0(t-t_0)}\widehat{V}e^{-iH_0(t-t_0)}$, which we expand in a basis of operators

再转换到相互作用绘景, 因此 $\mathcal{O}(t) = e^{iH_0(t-t_0)}\widehat{\mathcal{O}}e^{-iH_0(t-t_0)}$, 态演化仅由 $V(t) = e^{iH_0(t-t_0)}\widehat{V}e^{-iH_0(t-t_0)}$ 控制，我们将其在算子基下展开

$$V(t) = \sum_n A_n(t) \otimes B_n(t). \tag{21}$$

In interaction picture the evolution super-operator becomes $\mathcal{L}_t(\mathcal{O}) = -i[V(t), \mathcal{O}]$ and (19) can be fruitfully expanded in powers of $V(t)$.

在相互作用绘景下，演化超算子变为 $\mathcal{L}_t(\mathcal{O}) = -i[V(t), \mathcal{O}]$ ，方程 (19) 可以按 $V(t)$ 的幂次有效展开。

For our later purposes it suffices to work only to second order in V , in which case the kernel becomes $\mathcal{K} \simeq \mathcal{K}_2 = \mathcal{P}\mathcal{L}_t\mathcal{Q}\mathcal{L}_s\mathcal{P}$. Choosing an uncorrelated initial condition for the interaction-picture density matrix, $\rho(t_0) = \rho_A(t_0) \otimes \rho_B$, Eq. (19) reduces to the following approximate expression

对于我们后续的研究目的，只需计算到 V 的二阶项，此时核变为 $\mathcal{K} \simeq \mathcal{K}_2 = \mathcal{P}\mathcal{L}_t\mathcal{Q}\mathcal{L}_s\mathcal{P}$ 。对相互作用绘景下的密度矩阵选取无关联初条件 $\rho(t_0) = \rho_A(t_0) \otimes \rho_B$ 后，方程 (19) 简化为如下近似表达式

$$\begin{aligned}\partial_t\rho_A(t) = -i\sum_n [A_n(t), \rho_A(t)]\langle\langle B_n(t)\rangle\rangle \\ +(-i)^2\sum_{mn}\int_{t_0}^{t} \mathrm{d}s\{[A_m(t), A_n(s)\rho_A(s)]\langle\langle\delta B_m(t)\delta B_n(s)\rangle\rangle \\ -[A_m(t), \rho_A(s)A_n(s)]\langle\langle\delta B_n(s)\delta B_m(t)\rangle\rangle\} + \mathcal{O}\left(V^3\right),\end{aligned} \tag{22}$$

which introduces the notation $\langle\langle(\cdots)\rangle\rangle = \mathrm{tr}_B[(\cdots)\rho_B]$ for averages over sector B and $\delta B := B - \langle\langle B\rangle\rangle$. The implications of this equation when applied to gravitating systems are explored at some length below.

其中引入了记号 $\langle\langle(\cdots)\rangle\rangle = \mathrm{tr}_B[(\cdots)\rho_B]$ ，用于对区间 B 和 $\delta B := B - \langle\langle B\rangle\rangle$ 求平均。下文会较为详细地探讨该方程应用于引力系统时的相关结论。

So far the discussion has been general, but perhaps puzzling to find in a review volume on effective field theory. We next introduce a hierarchy of scales since this lies at the root of the simplicity that EFT methods exploit. In the present instance it is more useful to express this hierarchy in the time domain (rather than energy) and so we now assume sector B includes 'fast' degrees of freedom relative to a set of slower variables that are of interest in sector A . In particular, we assume the correlation functions $\langle\langle\delta B_n(t)\delta B_m(s)\rangle\rangle$ fall to zero once $t-s$ is much larger than a characteristic timescale, τ_c . A useful hierarchy arises if τ_c is much smaller than the timescale t_A over which the evolution of $\rho_A(t)$ is sought.

到目前为止我们的讨论都是一般性的，在有效场论的综述文集中出现这类内容或许令人困惑。接下来我们引入标度分层，因为这正是有效场论方法能够发挥简洁性的核心。在当前问题中，在时域而非能域表示这种分层更为有用，因此我们假设: 相对于区间 A 中我们关注的一组慢变量，区间 B 包含“快”自由度。特别地，我们假设关联函数 $\langle\langle \delta B_n(t)\,\delta B_m(s)\rangle\rangle$ 在 $t-s$ 远大于特征时标 τ_c 时衰减为零。如果 τ_c 远小于我们观测 $\rho_A(t)$ 演化所用的时标 t_A ，就会形成有用的标度分层。

Under these assumptions Eq. (22) becomes approximately local in time because the rest of the integrand varies much more slowly than $\langle\langle \delta B_n(t)\,\delta B_m(s)\rangle\rangle$ near $s=t$ and so can be Taylor expanded about $s=t$, with the logic that contributions of successive terms to the integral should be suppressed by powers of τ_c/t_A . Once this is done $\rho_A(t)$ (and its derivatives) can be factored out of the integral. In this case the evolution equation for ρ_A simplifies to

在这些假设下，式 (22) 近似为时间定域形式，因为被积函数的其余部分在 $s=t$ 附近的变化远慢于 $\langle\langle \delta B_n(t)\,\delta B_m(s)\rangle\rangle$ ，因此可以在 $s=t$ 附近做泰勒展开，其依据是，successive 项对积分的贡献会被 τ_c/t_A 的幂次压低。完成该操作后，$\rho_A(t)$ (及其导数) 可以从积分中提取出来。此时 ρ_A 的演化方程简化为

$$\partial_t\rho_A \simeq -i\left[\sum_n A_n\,\langle\langle B_n\rangle\rangle + \sum_{mn} h_{mn}A_mA_n, \rho_A\right] + \sum_{mn}\gamma_{mn}\left[A_n\rho_A A_m - \frac{1}{2}\{A_mA_n,\rho_A\}\right],$$

(23)

where the coefficients and operators on the right-hand side are all evaluated at a the same time, t , as for the left-hand side and

其中右侧所有系数和算符都和左侧一样，取同一时刻 t 的值，且

$$\left\{\begin{array}{lll}\gamma_{mn} & := & C_{mn} + C^*_{nm}\\ h_{mn} & := & \frac{1}{2i}\left(C_{mn} - C^*_{nm}\right)\end{array}\right. \quad \text{with } C_{mn}(t) := \int_{t_0}^{t} \mathrm{d}s\,\langle\langle \delta B_m(t)\,\delta B_n(s)\rangle\rangle,$$

(24)

where Hermiticity of the B_n ’ s implies $\gamma^*_{nm}=\gamma_{mn}$ and $h^*_{nm}=h_{mn}$.

其中 B_n 的厄米性给出 $\gamma^*_{nm}=\gamma_{mn}$ 和 $h^*_{nm}=h_{mn}$ 。

This is precisely the equation we would have obtained if the sector- B correlation function were approximately local in time,

这正是 sector- B 关联函数近似时间定域时我们会得到的方程,

$$\langle\langle \delta B_m(t)\,\delta B_n(s)\rangle\rangle \simeq C_{mn}(t)\,\delta(t-s)\,. \tag{25}$$

An approximate master equation of this type is called a Lindblad - or GKSL (Gorini, Kossakowski, Sudarshan, Lindblad) - equation [7,8]. The Hermiticity and positivity of γ_{mn} is crucial for ensuring that ρ_A remains hermitian and non-negative for all t , as is required for its eigenvalues to carry a probability interpretation.

这类近似主方程称为林德布拉德方程 (或称 GKSL 方程，得名于 Gorini、Kossakowski、Sudarshan、Lindblad)[7,8]。γ_{mn} 的厄米性与正定性对保证 ρ_A 对所有 t 都保持厄米性和非负性至关重要，这是其本征值具有概率诠释的要求。

Equation (23) is much easier to work with than (22) because it is Markovian in the sense that $\partial_t \rho_A(t)$ depends only on other variables at time t and not on the entire history of evolution prior to this time. We next expand on why this property also can be useful for extending to later times the predictions of perturbation theory.

式 (23) 比式 (22) 更容易处理，因为它是马尔可夫性的: $\partial_t \rho_A(t)$ 仅依赖于时刻 t 的其他变量，不依赖于该时刻之前的整个演化历史。我们接下来展开说明，为什么该性质对将微扰论的预测推广到更晚时间也有用。

Late-Time Failure of Perturbative Methods

微扰方法的后期失效

As mentioned earlier, whenever a system interacts with a persistent environment it is generic that perturbative methods eventually fail to accurately capture time evolution: $\exp\left[-i\int V\,\mathrm{d}t\right]$ eventually differs significantly from $1-i\int V\,\mathrm{d}t$. Yet we are often able to make reliable late-time predictions anyway, even when $\int V\,\mathrm{d}t$ is order unity. In section ”Secular Growth and the Minkowski Vacuum” we provide a few examples where this kind of ’secular growth’ of perturbative corrections actually arise in gravitational settings, but for now we just briefly review when and why this is possible and argue why an evolution like Eq. (23) can sometimes give reliable late-time predictions despite being derived from (22) (whose utility in the far future generically breaks down).

如前文所述，当系统与持续存在的环境发生相互作用时，微扰方法通常最终都无法准确描述时间演化: $\exp\left[-i\int V\,\mathrm{d}t\right]$ 最终会与 $1-i\int V\,\mathrm{d}t$ 产生显著差异。但即便 $\int V\,\mathrm{d}t$ 约为 1，我们往往仍能给出可靠的后期预测。在“长期增长与闵氏真空”一节中，我们会给出几个这类微扰修正的“长期增长”实际出现在引力场景中的例子，目前我们仅简要回顾这种情况何时会发生、为何会发生，并论述为什么类似式 (23) 的演化，尽管是从 (22) 推导而来 ((22) 的适用性在远未来通常会失效)，有时仍能给出可靠的后期预测。

Radioactive Decay

放射性衰变

Radioactive decay is instructive in this context because it provides a well-understood example where the apparent breakdown of perturbation theory at late times can be circumvented. Consider therefore an unstable parent particle that spontaneously decays (perhaps through the weak interactions) into a collection of daughter particles: $P \to D_1 + D_2 + \cdots$. In the simplest situations such decays can be computed perturbatively and arise - in the absence of a conservation law that forbids the decay - due to the existence of a nonzero matrix element $\langle D_1, D_2, \cdots |V| P\rangle \sim \; O(g)$ for some coupling $g \ll 1$. Standard expressions give the differential decay rate

在此背景下，放射性衰变很有启发意义，因为它提供了一个容易理解的例子，说明微扰论在晚时间的明显失效可以被绕开。来考虑一个不稳定母粒子自发衰变 (可能通过弱相互作用) 为一系列子粒子的过程: $P \to D_1 + D_2 + \cdots$ 。最简单的情况下，这类衰变可以微扰计算；不存在禁止该衰变的守恒律时，衰变发生是因为对耦合 $g \ll 1$ 存在非零矩阵元 $\langle D_1, D_2, \cdots |V| P\rangle \sim \; O(g)$ 。标准推导给出微分衰变率

$$\mathrm{d}\Gamma \propto |\langle D_1, D_2, \cdots |V| P\rangle|^2 = O\left(g^2\right), \tag{26}$$

showing that decays first arise at second order in the interaction responsible.

表明衰变效应首次出现在相互作用的二阶项中。

More subtle is the justification for the (survival) probability for a parent particle not to decay that follows the well-known exponential decay law,

更微妙的问题是如何推导师从母粒子不衰变的 (存活) 概率满足著名的指数衰变定律,

$$p(t) = e^{-\Gamma(t-t_0)}. \tag{27}$$

Equation (27) is experimentally verified to hold for times much longer than the decay’s mean lifetime, $\tau = 1/\Gamma$, and so for times $\Gamma(t-t_0) \gg 1$. Given that Γ is computed only to order g^2 how can (27) be regarded as more accurate than the strictly perturbative result

式 (27) 经实验验证，在远长于衰变平均寿命 $\tau = 1/\Gamma$ 的时间范围内成立，即对应时间 $\Gamma(t-t_0) \gg 1$ 。既然 Γ 仅计算到 g^2 阶，为何式 (27) 可以比严格微扰结果更精确?

$$p(t) \simeq 1 - \Gamma(t-t_0) + \cdots, \tag{28}$$

once $t - t_0 \gtrsim \tau$?

当 $t - t_0 \gtrsim \tau$ 时?

Exponential decays arise whenever the survival probability $p(t)$ is a solution to

只要存活概率 $p(t)$ 是下述方程的解，就会产生指数衰变

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\Gamma p \tag{29}$$

Although this evolution equation is consistent with (28) it has a broader domain of validity because it relies only on the likelihood of decays in any short interval dt being independent of decays in any other time windows. The value of Γ appearing in the exponential can be extracted from (28) because it agrees with (29) for small times, but once this is done the solutions to (29) can be trusted for much longer times (see Fig. 1).

尽管这个演化方程与 (28) 自洽，但它的有效范围更广，因为它仅依赖于一个性质: 任意短间隔 $\mathrm{d}t$ 内发生衰变的概率与其他时间窗内的衰变无关。指数中出现的 Γ 可以从式 (28) 得到，因为它在小时间下与式 (29) 一致；但确定这个值后，式 (29) 的解在长得多的时间下依然可信 (见图 1)。

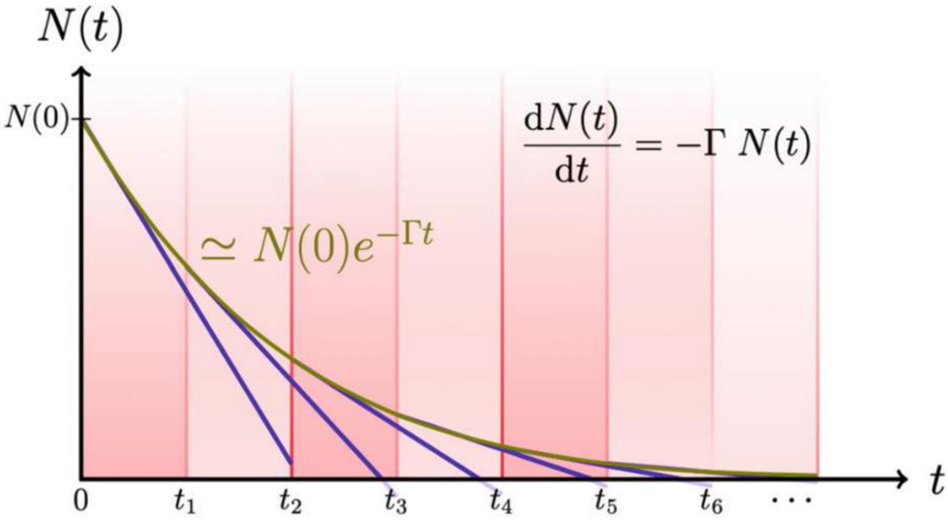

Fig. 1 Perturbation theory gives a survival probability that is linear in t , but does so over a tiny interval $t \in [0, t_1]$ for which $\Gamma t \ll 1$. Because the evolution equation for $N(t) = p(t)\, N_0$ implies $\mathrm{d}N/\mathrm{d}t$ is local in time it applies equally well for any interval (t_n, t_{n+1}) using perturbative methods, making its solutions valid over the union of all possible such intervals. The evolution equation stitches together the perturbative expressions to give the resummed solution $N(t) \simeq N(0)\, e^{-\Gamma t}$ out to very late times

图 1 微扰论给出的存活概率是 t 的线性函数，但仅在满足 $\Gamma t \ll 1$ 的极小间隔 $t \in [0, t_1]$ 内成立。由于 $N(t) = p(t)\, N_0$ 的演化方程要求 $\mathrm{d}N/\mathrm{d}t$ 是时间局域的，对任意间隔 (t_n, t_{n+1}) 都可以用微扰方法处理，因此演化方程的解在所有这类区间的并集上都有效。演化方程将微扰表达式拼接，得到一直延伸到极晚时间的重求和解 $N(t) \simeq N(0)\, e^{-\Gamma t}$

This is a very powerful line of argument, and when it works it allows working to all orders in $g^2 t$ without having to understand all observables at all orders in g . Use of the leading-order expression $\Gamma \sim O(g^2)$ when integrating (29) amounts to resumming all orders in $g^2 t$ while dropping terms like $g^4 t$ that involve extra powers of the interaction V without the corresponding extra powers of t . Integrating (29) using an order g^4 expression for Γ similarly gives a result valid to all orders in $g^4 t$ while dropping terms like $g^6 t$ and so on.

这是一条非常有力的论证思路，当它成立时，我们可以得到 $g^2 t$ 所有阶的结果，而不需要理解 g 所有阶的全部可观测量。对式 (29) 积分时使用领头阶表达式 $\Gamma \sim O(g^2)$ ，相当于对 $g^2 t$ 做所有阶重求和，同时舍弃 $g^4 t$ 这类项——这类项包含相互作用 V 的额外幂次，但没有 t 的对应额外幂次。使用 g^4 阶的 Γ 表达式积分式 (29)，同理可以得到在 $g^4 t$ 所有阶都成立的结果，同时舍弃 $g^6 t$ 这类项，以此类推。

The key to this argument is the derivation of an evolution equation like (29) that can have broader validity than a straight-up perturbative approach. This is possible because the evolution equation itself does not make explicit reference to the initial time and so could apply equally well for any initial starting point t_0 . The same argument would not be expected to be possible using (22), for instance, since although this equation is derived on very general grounds its right-hand side makes explicit reference to both t_0 and t . It could apply to the evolution given in (23) however - such as if C_{mn} defined in (24) were independent of t_0 - because this also makes explicit reference only to physics at time t (and not to quantities like t_0).

该论证的关键在于推导出如 (29) 式这类适用范围比直接微扰方法更广的演化方程。这之所以可行，是因为演化方程本身并未明确依赖初始时间，因此它可以同等适用于任意初始起点 t_0 。例如，我们无法指望用 (22) 式完成同样的论证，因为尽管该方程是基于非常普遍的前提推导得到的，但其右侧同时明确依赖 t_0 和 t 。不过该论证可以适用于 (23) 式给出的演化——例如当 (24) 式中定义的 C_{mn} 独立于 t_0 时——因为此时方程也仅明确涉及时刻 t 的物理量 (不涉及 t_0 这类量)。

Qubit Thermalization

量子比特热化

To make the above discussion concrete it is useful to examine an explicit example for which a Lindblad-type equation describes late-time behaviour. We also build in two features that are useful in some of the examples to follow: they involve thermal environments and reduced evolution is only sought for a simple two-level system (a qubit) for which all calculations can be made very explicit.

为了让上述讨论更具体, 我们来考察一个显式例子: 林德布拉德型方程描述了该例子的晚期行为。我们还引入了后续部分例子中会用到的两个特性: 例子涉及热环境，且我们仅需求解简单二能级系统 (量子比特) 的约化演化，所有计算都可以非常明确地进行。

To this end consider a two-level system (with levels split by energy ω) coupled to an environment (taken here to be a relativistic real massless scalar field) prepared in a thermal state with temperature T . The unperturbed Hamiltonian is therefore taken to be $H_0 = H_A \otimes I_B + I_A \otimes H_B$ where

为此，我们考虑一个二能级系统 (能级间距为能量 ω) 耦合到环境 (此处取相对论性无质量实标量场)，整个系统初态为温度 T 下的热态。因此未受扰哈密顿量取为 $H_0 = H_A \otimes I_B + I_A \otimes H_B$ ，其中

$$H_A = \frac{\omega}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ and } H_B = \frac{1}{2}\int \mathrm{d}^3x \left[(\partial_t\phi)^2 + (\nabla\phi)^2\right]. \tag{30}$$

Massive fields can be treated equally explicitly though involve somewhat more cumbersome expressions.

有质量场也可以做同样显式的处理，只是表达式会更繁琐一些。

The interaction-picture interaction linking these two systems is assumed to be

连接两个系统的相互作用绘景下相互作用假设为

$$V(t) = g\alpha(t) \otimes \phi[\mathbf{x}_0, t] \tag{31}$$

with the field evaluated at the qubit's (static) position $\mathbf{x}(t) = \mathbf{x}_0$ and

场在量子比特的 (静态) 位置 $\mathbf{x}(t) = \mathbf{x}_0$ 处取值，且

$$\alpha(t) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} e^{-i\omega t} + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} e^{i\omega t} \tag{32}$$

involves the qubit's raising and lowering operators in the interaction picture. The dimensionless coupling $g \ll 1$ is assumed small enough to justify perturbative methods. Putting ω into H_0 rather than V and working with non-degenerate perturbation theory assumes ω to be much larger than any perturbative field-induced shift in qubit energies, a condition made more explicit in (45) below.

涉及相互作用绘景下量子比特的升算符和降算符。无量纲耦合 $g \ll 1$ 假设足够小，满足微扰方法的适用条件。将 ω 放入 H_0 而非 V，并使用非简并微扰论，这要求 ω 远大于微扰场诱导的量子比特能级移动，该条件会在下文 (45) 式中明确给出。

The field ϕ is prepared in an initial thermal state

场 ϕ 的初态为热态

$$\rho_B = \frac{1}{Z} \exp\left[-\beta H_B\right], \tag{33}$$

with temperature $T = 1/\beta$ and $Z = \mathrm{tr}_B\left[\exp\left(-\beta H_B\right)\right]$ and so its Wightman function $W(\mathbf{x}, t; \mathbf{x}', t') := \langle\langle \phi(\mathbf{x}, t)\, \phi(\mathbf{x}', t') \rangle\rangle$ can be explicitly evaluated at coincident spatial points $\mathbf{x} = \mathbf{x}' = \mathbf{x}_0$. For a massless scalar field this gives

在温度 $T = 1/\beta$ 和 $Z = \mathrm{tr}_B\left[\exp\left(-\beta H_B\right)\right]$ 下，其威曼函数 $W(\mathbf{x}, t; \mathbf{x}', t') := \langle\langle \phi(\mathbf{x}, t)\, \phi(\mathbf{x}', t') \rangle\rangle$ 可以在重合空间点 $\mathbf{x} = \mathbf{x}' = \mathbf{x}_0$ 处显式计算得到。对于无质量标量场，可得

$$W(s) := W(\mathbf{x}_0, t + s; \mathbf{x}_0, t) = -\frac{1}{4\beta^2 \left[\sinh\left(\pi s/\beta\right) - i\varepsilon\right]^2}, \tag{34}$$

where $\varepsilon \to 0^+$ at the end of the calculation. Notice that $W(s)$ falls off exponentially once $s \gg \beta/\pi$ and diverges like $1/s^2$ as $s \to 0$.

其中计算末尾得到 $\varepsilon \to 0^+$ 。注意到当 $s \gg \beta/\pi$ 时，$W(s)$ 指数衰减；当 $s \to 0$ 时，$W(s)$ 按 $1/s^2$ 发散。

Substituting these expressions into the Nakajima-Zwanzig equation (22) and eliminating $\rho_{\downarrow\downarrow}$ and $\rho_{\downarrow\uparrow}$ using $\mathrm{tr}\, \rho = 1$ and $\rho = \rho^\dagger$ shows the diagonal and off-diagonal components of ρ_A evolve independent of one another at $O\left(g^2\right)$,

将这些表达式代入中岛-赞奇方程 (22)，利用 $\mathrm{tr}\, \rho = 1$ 和 $\rho = \rho^\dagger$ 消去 $\rho_{\downarrow\downarrow}$ 和 $\rho_{\downarrow\uparrow}$ 后，可以得到 $O\left(g^2\right)$ 处 ρ_A 的对角元和非对角元独立演化，

$$\frac{\partial \rho_{\uparrow\uparrow}}{\partial t} = g^2 \int_{-t}^{t} \mathrm{d}s W(s)\, e^{-i\omega s} - 4g^2 \int_0^t \mathrm{d}s\, \mathrm{Re}\left[W(s)\right] \cos(\omega s)\, \rho_{\uparrow\uparrow}(t - s), \tag{35}$$

and

且

$$\frac{\partial \rho_{\uparrow\downarrow}}{\partial t} = -2g^2 \int_0^t \mathrm{d}s \, \mathrm{Re}\left[W(s)\right] e^{i\omega s} \rho_{\uparrow\downarrow}(t-s) \tag{36}$$

$$+2g^2 e^{2i\omega t} \int_0^t \mathrm{d}s \, \mathrm{Re}\left[W(s)\right] e^{-i\omega s} \rho^*_{\uparrow\downarrow}(t-s),$$

where we assume uncorrelated initial conditions $\rho(t=0) = \rho_0 \otimes \rho_B$ with ρ_0 (so far) unspecified.

其中我们假设初始条件无关联 $\rho(t=0) = \rho_0 \otimes \rho_B$，且 ρ_0 (到目前为止) 未指定。

If we choose the initial condition $\rho_0 = |\downarrow\rangle\langle\downarrow|$ then both $\rho_{\uparrow\uparrow}$ and $\rho_{\uparrow\downarrow}$ are at most $O(g)$ and so can be dropped on the right-hand side if we strictly work only to $O\left(g^2\right)$. The resulting expressions reduce to $\partial_t \rho_{\uparrow\downarrow} \lesssim O\left(g^3\right)$ and

如果我们选取初始条件 $\rho_0 = |\downarrow\rangle\langle\downarrow|$，那么 $\rho_{\uparrow\uparrow}$ 和 $\rho_{\uparrow\downarrow}$ 都最多是 $O(g)$ 量级，如果我们严格只计算到 $O\left(g^2\right)$ 阶，就可以将它们从方程右侧舍去。最终表达式简化为 $\partial_t \rho_{\uparrow\downarrow} \lesssim O\left(g^3\right)$ 和

$$\frac{\partial \rho_{\uparrow\uparrow}}{\partial t} \simeq g^2 \int_{-t}^t \mathrm{d}s W(s) e^{-i\omega s}, \tag{37}$$

showing how the interaction with the thermal field starts to occupy the qubit's excited state. Because $W(s)$ is exponentially peaked around $s = 0$ with width β the right-hand side of this equation quickly approaches the t -independent constant $g^2 R(\omega)$ for $t \gg \beta$, where

展示与热场的相互作用如何开始占据量子比特的激发态。由于 $W(s)$ 在 $s = 0$ 附近呈指数峰形，宽度为 β，该方程的右侧在 $t \gg \beta$ 条件下会快速趋近于与 t 无关的常数 $g^2 R(\omega)$，其中

$$R(\omega) = \int_{-\infty}^{\infty} \mathrm{d}s W(s) e^{-i\omega s} = \frac{1}{2\pi} \frac{\omega}{e^{\beta\omega} - 1}. \tag{38}$$

Integrating (37) to obtain $\rho_{\uparrow\uparrow}(t)$ then gives the characteristic linear dependence on t that signals a breakdown of perturbation theory at times $t \gtrsim \tau_p := \left[g^2 R(\omega)\right]^{-1}$. Notice that this breakdown occurs at a time depending sensitively on $\beta\omega$, with $\tau_p \simeq 2\pi\beta/g^2$ when $\beta\omega \ll 1$ and $\tau_p \simeq \left(2\pi/g^2\omega\right) e^{\beta\omega}$ when $\beta\omega \gg 1$.

对 (37) 积分得到 $\rho_{\uparrow\uparrow}(t)$ 后，可得出其对 t 的特征线性依赖关系，这标志着微扰论在 $t \gtrsim \tau_p := \left[g^2 R(\omega)\right]^{-1}$ 时刻失效。请注意，失效发生的时间对 $\beta\omega$ 非常敏感，当 $\beta\omega \ll 1$ 时 $\tau_p \simeq 2\pi\beta/g^2$，当 $\beta\omega \gg 1$ 时 $\tau_p \simeq \left(2\pi/g^2\omega\right) e^{\beta\omega}$。

To learn the behaviour of $\rho_A(t)$ for $t \sim \tau_p$ we return to equations (35) or (36). Progress can be made if the rest of the integrand varies only over times very long compared with the support of $W(\tau)$, in which case we can expand terms like

为了得到 $\rho_A(t)$ 在 $t \sim \tau_p$ 条件下的行为，我们回到方程 (35) 或 (36)。若被积函数的其余部分仅在远长于 $W(\tau)$ 支集的时间尺度上变化，我们就可以继续推导，此时可对如下形式的项进行展开:

$$\alpha(t-s)\,\rho_{ab}(t-s) \simeq \alpha(t)\,\rho_{ab}(t) - s[\partial_t(\alpha\rho_{ab})]_{s=0} + \cdots, \tag{39}$$

and integrate the result term by term. Here $\alpha(t)$ is the interaction-picture matrix appearing in (31) whose presence is responsible for oscillatory factors involving $e^{\pm i\omega t}$. In particular, the combination $\alpha\rho_{ab}$ can only vary slowly over times of order β if $\beta\omega \ll 1$, which we henceforth assume.

再对结果逐项积分。其中 $\alpha(t)$ 是出现在 (31) 中的相互作用绘景矩阵，其存在导致了含 $e^{\pm i\omega t}$ 的振荡因子。特别地，若 $\beta\omega \ll 1$ ，则组合 $\alpha\rho_{ab}$ 仅在量级为 β 的时间上缓慢变化，我们此后将此作为假设。

Dropping all but the first term of the expansion (39) in (35) and (36) leads to Markovian evolution of the form of (23), which for $\rho_{\uparrow\uparrow}$ and $\rho_{\uparrow\downarrow}$ become

在 (35) 和 (36) 中仅保留展开式 (39) 的第一项，可得到形式为 (23) 的马尔可夫演化，在 $\rho_{\uparrow\uparrow}$ 和 $\rho_{\uparrow\downarrow}$ 条件下，该演化变为

$$\frac{\partial\rho_{\uparrow\uparrow}}{\partial t} \simeq g^2 R_0\left[1 - 2\rho_{\uparrow\uparrow}(t)\right], \tag{40}$$

and

和

$$\frac{\partial\rho_{\uparrow\downarrow}}{\partial t} \simeq -g^2 R_0\left[\rho_{\uparrow\downarrow}(t) + e^{2i\omega t}\rho^*_{\uparrow\downarrow}(t)\right], \tag{41}$$

with $R_0 = (2\pi\beta)^{-1}$ the $\beta\omega \ll 1$ limit of the function $R(\omega)$ given in (38). Because (40) makes no reference to the initial time, its domain of validity is broader than straight-up perturbation theory would be, allowing its solutions to be trusted at later times through the same reasoning as used above for exponential decays.

其中 $R_0 = (2\pi\beta)^{-1}$ 是 (38) 给出的函数 $R(\omega)$ 的 $\beta\omega \ll 1$ 极限。由于 (40) 不依赖初始时间，它的适用范围比直接微扰论更广，按照前文对指数衰减所用的相同推论，其解在更晚的时间依然可靠。

Integrating leads to the solutions

积分后得到解:

$$\rho_{\uparrow\uparrow}(t) = \frac{1}{2} + \left[\rho_{\uparrow\uparrow}(0) - \frac{1}{2}\right]e^{-t/\xi_{d0}}, \tag{42}$$

and

和

$$\rho_{\uparrow\downarrow}(t) = \left[\rho_{\uparrow\downarrow}(0) + \frac{ig^2R_0}{2\omega}\left(1 - e^{2i\omega t}\right)\rho^*_{\uparrow\downarrow}(0)\right]e^{-t/\xi_{c0}}, \tag{43}$$

with [3]

其中 [3]

$$\xi_{c0} = 2\xi_{d0} = \frac{1}{g^2 R_0} = \frac{2\pi\beta}{g^2}. \tag{44}$$

For small times Eq. (42) agrees with the $\beta\omega \to 0$ limit of (37) and describes initial qubit excitation due to the presence of the field. But Eqs. (42) and (43) also apply for $t \sim \xi_{d0}$ and describe late-time relaxation towards a steady state in which the qubit becomes completely mixed (more about which below): $\rho_A \to \rho_\infty = \mathrm{diag}\left(\frac{1}{2}, \frac{1}{2}\right)$. Notice that the late-time relaxation rate ξ_{d0} differs from the timescale $1/\left(g^2 R_0\right)$ that describes the early-time perturbative excitation rate out of the ground state - given by (37) and (38) - even when $\beta\omega \to 0$.

对于小时间，式 (42) 与 (37) 的 $\beta\omega \to 0$ 极限一致，描述了场存在导致的量子比特初始激发。但式 (42) 和 (43) 同样适用于 $t \sim \xi_{d0}$ ，描述量子比特向稳态弛豫的晚期过程，最终量子比特变为完全混合态 (下文将进一步讨论): $\rho_A \to \rho_\infty = \mathrm{diag}\left(\frac{1}{2}, \frac{1}{2}\right)$。请注意，即使在 $\beta\omega \to 0$ 条件下，晚期弛豫率 ξ_{d0} 也不同于描述早期微扰过程中基态激发速率的时间尺度 $1/\left(g^2 R_0\right)$ ——后者由 (37) 和 (38) 给出。

We can now better quantify the size of the contributions to (35) and (36) by the subdominant terms in the expansion (39). These should be suppressed by powers of either $\beta\omega/\pi$ or $\beta/(\pi\xi_{d0}) = 2g^2\beta R_0/\pi \simeq (g/\pi)^2$ and so are indeed negligible for perturbatively small g provided $\beta\omega \ll 1$ as well. We may also better quantify the lower limit on ω alluded to earlier that is required by our use of nongenerate perturbation theory (which assumes the effects of $V(t)$ are perturbatively small relative to H_A). This requires $\omega \gg g^2 R_0 \simeq g^2/(2\pi\beta)$ and so combining all requirements we find that late-time evolution becomes Markovian and reliably computable within the regime

现在我们可以借助展开式 (39) 中的次要项，更准确地量化对 (35) 和 (36) 的贡献大小。这些贡献应当会被 $\beta\omega/\pi$ 或 $\beta/(\pi\xi_{d0}) = 2g^2\beta R_0/\pi \simeq (g/\pi)^2$ 的幂次压低，因此只要满足 $\beta\omega \ll 1$ ，对于微扰小量 g 而言，这些贡献确实可以忽略。我们还可以更准确地量化前面提到的 ω 的下限，该下限是我们使用非简并微扰论所要求的——非简并微扰论假设 $V(t)$ 的效应相对于 H_A 是微扰小量。这要求满足 $\omega \gg g^2 R_0 \simeq g^2/(2\pi\beta)$ ，因此结合所有条件我们发现，在该适用范围内，晚期演化是马尔可夫过程，并且可以可靠计算

$$1 \gg \omega\beta \gg \frac{g^2}{2\pi}. \tag{45}$$

[3] This result turns out to saturate a general upper bound $\xi_{c0} < 2\xi_{d0}$.

[3] 该结果恰好达到了广义上界 $\xi_{c0} < 2\xi_{d0}$ 的饱和。

In the above treatment predictions acquire corrections as powers of $\beta\omega$ once the higher-order contributions in (39) are included. We note in passing that some of this ω -dependence can be explored if only the density matrix is Taylor expanded: $\rho_{ab}(t-s) \simeq \rho_{ab}(t) - s\partial_t\rho_{ab}(t) + \cdots$ without also expanding $\alpha(t-s)$ as in (39). Doing so in (35) modifies (40) to

在上述处理中，当我们纳入 (39) 的高阶贡献后，预言会得到以 $\beta\omega$ 为幂次的修正。我们顺便指出，如果仅对密度矩阵做泰勒展开: 即只展开 $\rho_{ab}(t-s) \simeq \rho_{ab}(t) - s\partial_t \rho_{ab}(t) + \cdots$ ，不按照 (39) 同时展开 $\alpha(t-s)$ ，就可以探究部分这种对 ω 的依赖关系。将这种做法代入 (35)，会将 (40) 修改为

$$\frac{\partial \rho_{\uparrow\uparrow}}{\partial t} \simeq g^2 R - 2g^2 C \rho_{\uparrow\uparrow}(t), \tag{46}$$

where $R(\omega)$ is given by (38) and the new function $C(\omega)$ is defined by

其中 $R(\omega)$ 由 (38) 给出，新函数 $C(\omega)$ 定义为

$$C(\omega) := \int_{-\infty}^{\infty} \mathrm{d}s \, [\mathrm{Re}\, W(s)] \cos(\omega s) = \frac{\omega}{4\pi} \coth\left(\frac{\beta\omega}{2}\right). \tag{47}$$

This does not mean we get to drop the requirement that $\beta\omega$ must be small since this is still required to ensure that subdominant terms of the expansion of $\rho_{ab}(t-s)$ are small relative to the leading contribution [9].

这并不意味着我们可以取消 $\beta\omega$ 必须为小量的要求，因为要保证 $\rho_{ab}(t-s)$ 展开的次要项相对于领头贡献很小，该条件仍然是必要的 [9]。

The solution to (46) with R and C given by (38) and (47) is

方程 (46) 在 R 和 C 分别由 (38) 和 (47) 给出时的解为

$$\rho_{\uparrow\uparrow}(t) = \frac{1}{e^{\beta\omega}+1} + \left[\rho_{\uparrow\uparrow}(0) - \frac{1}{e^{\beta\omega}+1}\right] e^{-t/\xi_d}, \tag{48}$$

with

其中

$$\xi_d = \frac{1}{2g^2 C(\omega)} = \frac{2\pi}{g^2\omega} \tanh\left(\frac{\beta\omega}{2}\right). \tag{49}$$

We see from this that the late-time evolution of ρ_A describes thermalization; the qubit relaxes towards the thermal state

由此我们可以看到，ρ_A 的晚期演化描述了热化过程；量子比特会弛豫到热态

$$\rho_{\text{th}} := \begin{bmatrix} \frac{1}{1+e^{\beta\omega}} & 0 \\ 0 & \frac{1}{1+e^{-\beta\omega}} \end{bmatrix} = \begin{bmatrix} e^{-\beta\omega} & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{1+e^{-\beta\omega}} \tag{50}$$

that shares the field’s temperature. ρ_{th} is the unique static solution to (46) - and to (52) below - and so is the state to which solutions relax.

该热态与场具有相同温度。ρ_{th} 是 (46) 以及下文 (52) 唯一的静态解，因此它是所有解最终弛豫到的状态。

A similar story goes through for $\rho_{\uparrow\downarrow}$, with Taylor expansion of $\rho_{\uparrow\downarrow}(t-s)$ again removing the history-dependence of Eq. (36), but with a few complications. The complications arise because the equation obtained after Taylor expansion involves a new function

> 对于 $\rho_{\uparrow\downarrow}$ 也有类似的结论，对 $\rho_{\uparrow\downarrow}(t-s)$ 做泰勒展开同样可以消除式 (36) 的历史依赖性，但会带来一些额外的复杂问题。这些问题来自泰勒展开后得到的方程包含一个新函数

$$\Delta(\omega) \;:=\; 2\int_0^{\infty} \mathrm{d}s\, \mathrm{Re}\left[W(s)\right] \sin(\omega s) \tag{51}$$

as well as the function $C(\omega)$ encountered in (47). This new function causes problems partly because it diverges in the $s \to 0$ part of the integration region. (The function

> 以及我们在 (47) 中遇到过的函数 $C(\omega)$。这个新函数会引发问题，部分原因是它在积分区域的 $s \to 0$ 部分发散。(该函数

C does not similarly diverge because of the Wightman function’s $i\varepsilon$ factor seen in Eq. (34).) This is an ultraviolet divergence, and because it appears together with the qubit frequency ω it can be renormalized into the physical frequency: $\omega_R = \omega + g^2\Delta$.

> C 不会发生类似发散，这得益于式 (34) 中怀特曼函数的 $i\varepsilon$ 因子。) 这是一个紫外发散，由于它和量子比特频率 ω 一同出现，因此可以将其重整化到物理频率中: $\omega_R = \omega + g^2\Delta$。

The second complication arises because the finite part of Δ is proportional to ω in the limit $\beta\omega \ll 1$ and as a result is systematically smaller in this limit than is $C \propto \beta^{-1}$, requiring Δ to be neglected relative to C . If these Δ -dependent terms are mistakenly kept then comparing (46) and (52) with the general Lindblad form (23) shows that the matrix γ_{mn} that results is not positive. [4] Careful treatment shows that the apparent negative eigenvalues are always spurious if one religiously restricts to the domain of validity of all approximations (as must be the case).

> 第二个复杂情况出现的原因是，在极限 $\beta\omega \ll 1$ 下，Δ 的有限部分与 ω 成正比，因此在该极限下，它系统地小于 $C \propto \beta^{-1}$，这就要求相对于 C 忽略 Δ。如果错误地保留这些依赖于 Δ 的项，那么将 (46) 和 (52) 与一般的林德布拉德形式 (23) 进行比较会发现，得到的矩阵 γ_{mn} 不是正定的。[4] 仔细处理表明，如果严格限制在所有近似的有效范围内 (情况必然如此)，那么明显的负特征值总是虚假的。

Keeping these points in mind, the resulting leading evolution equation again has the Lindblad form, (23),

> 牢记这些要点后，最终得到的领头阶演化方程仍然满足林德布拉德形式，即式 (23),

$$\frac{\partial \rho_{\uparrow\downarrow}}{\partial t} \simeq -g^2 C \rho_{\uparrow\downarrow}(t) + g^2 C e^{2i\omega t} \rho^*_{\uparrow\downarrow}(t)\,, \tag{52}$$

with solution

> 其解为

$$\rho_{\uparrow\downarrow}(t) = \left[\rho_{\uparrow\downarrow}(0) + \frac{ig^2 C}{2\omega}\left(1 - e^{2i\omega t}\right)\rho^*_{\uparrow\downarrow}(0)\right] e^{-t/\xi_c}, \tag{53}$$

where $\xi_c = 2\xi_d$ and we drop the subscript ' R ' on ω_R .

其中 $\xi_c = 2\xi_d$ ，我们去掉了 ω_R 的下标“R”。

We learn two general things from this example. First, open systems can exhibit phenomena not seen in isolated quantum systems, such as the evolution from pure to mixed states that underlies the processes of decoherence and thermalization. Second, although straight-up perturbation in g fails to reliably capture evolution at late times where g^2t cannot be neglected, this failure can under some circumstances be resummed to give reliable results that are valid to all orders in g^2t . In this example the late-time evolution can be inferred because the full Nakajima-Zwanzig evolution becomes well-described by an approximate Lindblad equation that expresses how very slow evolution compared with the environment's typical correlation time can become Markovian (and so simpler). Solutions to the resulting Lindblad equation can be trusted at late times if its perturbative derivation works equally well in any small time interval.

我们从这个例子中得到两个一般性结论: 第一，开放系统可以展现出孤立量子系统没有的现象，例如退相干和热化过程背后从纯态到混态的演化。第二，虽然在 g^2t 不可忽略的长时尺度下，直接对 g 做微扰无法可靠描述演化，但在某些条件下，这种失效可以通过重求和得到对 g^2t 所有阶都成立的可靠结果。在这个例子中，我们可以推导出长时演化，因为完整的中岛-赞奇演化可以被一个近似林德布拉德方程很好地描述；该方程体现出，和环境典型关联时间相比极慢的演化可以成为马尔可夫过程 (因此更简单)。只要其微扰推导在任意小时间区间内都同样成立，得到的林德布拉德方程的解在长时尺度下就是可信的。

Secular Growth for Thermal Fields

热场的长期增长

Although it is generic that secular growth can arise for open quantum systems, does it actually arise for gravitating systems with horizons? The complete answer to this is not known because in many systems (such as black holes) the required calculations have not yet been completely performed. But the oft-remarked similarity between systems with horizons and thermal systems provides strong circumstantial evidence that secular growth is as ubiquitous as it is for thermal systems.

长期增长普遍可以出现在开放量子系统中，但对于带视界的引力系统，它真的会发生吗? 目前这个问题没有完整答案，因为在许多系统 (比如黑洞) 中，所需的计算尚未完全完成。但人们常注意到的带视界系统与热系统之间的相似性，提供了强有力的间接证据，表明长期增长在这里和在热系统中一样普遍存在。

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[4] Similar issues also arise in optics where a laser plays the role of the environment, and is tuned to a frequency close to ω , which is not small. In these applications a sensible Lindblad form is instead obtained only after performing a "rotating wave" approximation that averages over the fast oscillations. Such steps are not required in the applications considered here.

[4] 类似问题也出现在光学中: 激光扮演环境的角色，且被调谐到接近 ω 的频率，这个频率不小。在这类应用中，只有在执行了对快振荡做平均的“旋转波”近似后，才能得到合理的林德布拉德形式。本文讨论的应用中不需要这类步骤。

This section describes a simple example of this that is useful for the purposes of later comparison. To this end consider the same real scalar field prepared in a thermal state that was examined above as the environment with which a qubit interacted. But this time we ignore the qubit sector completely and instead study scalar field self-interactions as expressed by the action

本节描述一个此类的简单例子，便于后续比较。为此，我们考虑之前已经分析过的、制备在热态的实标量场，它作为量子比特相互作用的环境。但这次我们完全忽略量子比特部分，转而研究由作用量描述的标量场自相互作用:

$$S=-\int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\upsilon}\partial_\mu\phi\partial_\upsilon\phi+\frac{\lambda}{4}\phi^4\right], \tag{54}$$

with coupling λ chosen to be small and our interest lies in the validity of perturbative calculations in powers of λ . The gravitational field is included for future purposes though the appearance of the metric $g_{\mu\upsilon}$, but in this section we work purely in flat Minkowski space. In a Hamiltonian formalism this amounts to replacing the operator H_B of (30) with $H_{\text{tot}}=H_B+H_{\text{int}}$ where the self-interaction term is $H_{\text{int}}=\frac{1}{4!}\int d^3x\lambda\phi^4$. In terms of this the field’s thermal state is $-$cf . Eq. (33):

其中耦合 λ 取为小量，我们关注的是按 λ 幂次展开的微扰计算的有效性。引力场是为后续研究引入的，通过度规 $g_{\mu\upsilon}$ 体现，但本节我们纯在平直闵氏空间中工作。在哈密顿形式体系中，这相当于将 (30) 的算符 H_B 替换为 $H_{\text{tot}}=H_B+H_{\text{int}}$ ，其中自相互作用项为 $H_{\text{int}}=\frac{1}{4!}\int d^3x\lambda\phi^4$ 。用该形式表示，场的热态为 $-$cf 。式 (33):

$$\rho=\frac{1}{Z}\exp\left[-\beta H_{\text{tot}}\right] \tag{55}$$

with $\beta=1/T$ the inverse temperature and Z chosen as usual to ensure $\text{Tr}\,\rho=1$.

其中 $\beta=1/T$ 是逆温度，Z 按常规选取以保证 $\text{Tr}\,\rho=1$ 。

We seek an example of how secular growth arises for this scalar field system and so identify a quantity whose $O(\lambda)$ correction is a growing function of time. We follow [10] and use the Feynman correlation function $G(x;y)=\langle\mathcal{T}\phi(x)\phi(y)\rangle$ as our example, where $\mathcal{T}$ denotes time-ordering and the average is taken using the thermal state (55). To study secular evolution we compute order-λ corrections to $G(x,y)$, and because we wish time-dependence to be explicit we work in the real-time formalism computed within the Schwinger-Keldysh (or ”in-in”) framework [11,12].

我们要找一个实例，说明长期增长如何在这个标量场系统中产生，因此我们要找出一个量，它的 $O(\lambda)$ 修正为时间的增长函数。我们遵循文献 [10]，以费曼关联函数 $G(x;y) = \langle \mathcal{T}\phi(x)\phi(y)\rangle$ 为例，其中 $\mathcal{T}$ 表示时序，平均是利用热态 (55) 进行的。为了研究长期演化，我们计算对 $G(x,y)$ 的 λ 阶修正，并且为了显式体现时间依赖性，我们采用施温格-凯尔迪什 (或称“in-in”) 框架下的实时形式体系 [11,12]。

The leading correction in this case comes from the tadpole graph of Fig. 2. The loop part of this graph turns out to be position-independent and diverges in the ultraviolet (UV) in a temperature-independent way. Because it is temperature independent the UV divergence can be renormalized in the same way as at zero temperature, by choosing a mass counter-term to ensure that the renormalized zero-temperature mass remains zero. Once this is done, evaluation of the loop subgraph within Fig. 2 using the thermal state gives the finite $O(\lambda)$ temperature-dependent mass shift

这种情况下的领头阶修正来自图 2 的蝌蚪图。该图的圈部分与位置无关，且在紫外 (UV) 出现与温度无关的发散。由于发散与温度无关，紫外发散可以用零温下相同的方式重整化: 选取质量抵消项，保证重整化后的零温质量保持为零。完成这一步后，利用热态计算图 2 中的圈子图，会得到有限的、依赖温度的 $O(\lambda)$ 质量偏移:

$$\delta m_T^2 = \frac{\lambda T^2}{4} = \frac{\lambda}{4\beta^2}. \tag{56}$$

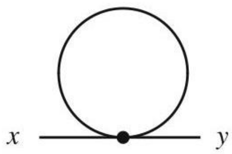

Fig. 2 Feynman graph giving the leading one-loop 'tadpole' correction to the scalar propagator. The result from this graph must be summed with the tree-level two-point counter-term graph

图 2 对标量传播子给出领头阶单圈“蝌蚪”修正的费曼图。该图的结果需要与树级二阶抵消项图求和

Using this self-energy in the remainder of the graph and evaluating the result at zero spatial separation, $\mathbf{y} = \mathbf{x}$ leads to the following form in the limit of large time difference $y^0 - x^0 = t$:

将这个自能用于图的其余部分，并在零空间分离处求值，$\mathbf{y} = \mathbf{x}$ 在大时间差 $y^0 - x^0 = t$ 极限下给出如下形式:

$$G_{\text{tad}}\left(x^0, \mathbf{x}; x^0 + t, \mathbf{x}\right) \simeq \frac{\delta m_T^2 T t}{8\pi} + \cdots = \frac{\lambda T^3 t}{32\pi} + \cdots, \tag{57}$$

where the ellipses denote terms that grow more slowly than linearly for large t . Behold secular growth: for sufficiently large t the correction to $G(x,y)$ is not small no matter how small λ is chosen to be.

其中省略号代表对于大 t 增长比线性更慢的项。来看长期增长: 无论 λ 选得多么小，对于足够大的 t ，对 $G(x,y)$ 的修正都不会很小。

Secular growth is related to (but not identical with) the infrared divergences that can arise when performing loops because both involve intermediate states with arbitrarily small frequency. This connection schematically arises because unusual growth at large times in a correlation function is usually related to singular behaviour for the small-frequency part of its Fourier transform, and such singularities can cause loop integrals to diverge because of contributions near $\omega = 0$. It is useful to further explore this connection since the resummation method used to handle IR divergences suggests how secular effects might also be resummed (at least in this particular example).

长期增长与做圈图计算时可能出现的红外发散相关 (但二者并不相同)，因为二者都涉及频率任意小的中间态。这种关联的概因在于: 关联函数在大时间处的异常增长通常与其傅里叶变换小频率部分的奇异行为相关，而此类奇异会导致圈积分因 $\omega = 0$ 附近的贡献发散。进一步探究这种关联很有意义，因为处理红外发散的重求和方法提示我们长期效应也可以如何进行重求和 (至少在这个特定例子中)。

Although the graph of Fig. 2 itself is infrared finite when the particles in the loop are massless, the same is not true for the higher loop 'cactus' graph of Fig. 3. The dangerous part of this graph for small k comes from the two propagators in the bottom loop which make it diverge logarithmically $\propto \int \mathrm{d}^4k/\big(k^2\big)^2$ even at zero temperature. By contrast, the bottom loop instead diverges like a power of the IR cutoff at finite temperature because of the singularity of the Bose-Einstein distribution [5]$n_B(k) = \big(e^{k/T} - 1\big)^{-1} \simeq T/k$ for $k \ll T$ contributing a factor

即使图 2 的图本身在圈中粒子为无质量时是红外有限的，图 3 的高阶圈“仙人掌”图却并非如此。该图对小 k 的危险部分来自底圈的两个传播子，它使得该图即使在零温下也会对数发散 $\propto \int \mathrm{d}^4k/\big(k^2\big)^2$ 。相比之下，有限温度下底圈会因玻色-爱因斯坦分布 [5]$n_B(k) = \big(e^{k/T} - 1\big)^{-1} \simeq T/k$ 的奇异而按红外截断的幂次发散，因为 $k \ll T$ 贡献了一个因子

$$\text{lower loop} \propto \frac{\lambda T}{\omega_{IR}}\,, \tag{58}$$

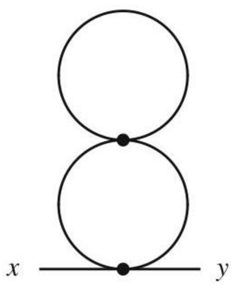

Fig. 3 Feynman graph giving a subleading two-loop 'cactus' correction to the scalar propagator. Although this is not the only two-loop contribution, it is noteworthy because of the power-law IR divergences it acquires due to the singularity of the Bose-Einstein distribution at small momenta

图 3 给标量传播子带来次领头双圈“仙人掌”修正的费曼图。虽然这不是唯一的双圈贡献，但它值得注意，因为小动量处玻色-爱因斯坦分布的奇异会让它获得幂律红外发散

where ω_{IR} is an IR cutoff.

其中 ω_{IR} 是红外截断。

Notice that the zero-temperature limit of the entire graph of Fig. 3 (including the IR-divergent part) precisely cancels with the graph where the upper loop is replaced by the mass counter-term once this counter-term is chosen to ensure $m^2 = 0$ at zero temperature. The same cancellation does not also occur at finite temperature because the counter-term only cancels the zero-temperature part of the top loop's contribution.

注意，一旦选择质量 counter-term 保证零温下的 $m^2 = 0$，图 3 整个图 (含红外发散部分) 的零温极限会与将上圈替换为该质量 counter-term 的图精确抵消。有限温度下不会发生同样的抵消，因为 counter-term 仅抵消顶圈贡献的零温部分。

Having Fig. 3 be proportional to (58) means that frequencies $\omega \leq \lambda T$ contribute unsuppressed relative to Fig. 2 because the large factor of T/ω_{IR} compensates for the small factor λ. The same is true for graphs with multiple bubbles added on top of one another since each extra bubble contributes an additional factor of (58). This signals a breakdown of perturbative methods since having λ be small no longer ensures that graphs with additional divergent higher loops come with the penalty of a small loop-counting parameter and suggests rethinking the split within the total Hamiltonian between H_0 and H_{int} to obtain a better-converging expansion.

图 3 正比于 (58) 意味着，相比于图 2，频率 $\omega \leq \lambda T$ 的贡献没有被压低，因为大因子 T/ω_{IR} 补偿了小因子 λ。对于多个泡相互叠加的图也是如此，因为每多一个泡就会多贡献一个 (58) 因子。这标志着微扰方法失效: 因为 λ 为小不再能保证，额外发散高阶圈的图会带有小圈计数参数的惩罚，这提示我们重新思考总哈密顿量中 H_0 与 H_{int} 的划分，以得到收敛性更好的展开。

The particular divergent contributions coming from IR-divergent bubble graphs are indeed famously resummed by moving the temperature-dependent mass into the unperturbed Hamiltonian - i.e. by adding and subtracting the temperature-dependent mass shift δm_T^2 and putting $m^2 + \delta m_T^2$ into H_0. In this case all internal lines represent massive states and the new $\delta m_T^2 = 0$ because the self-energy graphs systematically cancel with the corresponding graphs with the final bubble replaced by new mass counter-term, a well-known 'hard-thermal-loop' resummation [13, 14]. [6] This suggests that secular growth might similarly be resummed by perturbing around the full temperature-dependent mass.

源自红外发散泡图的特殊发散贡献确实可以通过将温度依赖质量移到未受扰哈密顿量中完成重求和，也就是通过加减温度依赖质量移动 δm_T^2，并将 $m^2 + \delta m_T^2$ 放入 H_0。在这种情况下所有内线都代表有质量态，且得到了新的 $\delta m_T^2 = 0$，因为自能图会系统地与最终泡替换为新质量 counter-term 的对应图抵消，这就是著名的“硬热圈”重求和 [13,14]。[6] 这提示我们，长期增长也可以通过围绕全温度依赖质量做扰动来类似地重求和。

Influence Functionals

影响泛函

An alternative approach to open systems uses the Feynman-Vernon influence functional [15], which is the path-integral version of the Hamiltonian evolution story told above for operators (see [2, 16 – 18] for textbook descriptions of this technique and [19] for a non-exhaustive list of their early use in general non-equilibrium and gravitational settings). One of their advantages is they are easily adapted to focus directly on correlation functions - as opposed to the reduced density matrix - and so can often allow one to cut directly to the chase when computing observables.

开放系统的另一种方法采用费曼-弗农影响泛函 [15]，它是上文对算符讲述的哈密顿量演化故事的路径积分版本 (该技术的教材描述见 [2, 16 − 18]，其早期在一般非平衡和引力背景下应用的非详尽列表见 [19])。它的一个优势是很容易调整为直接关注关联函数，而非约化密度矩阵，因此在计算可观测量时往往能直接切入正题。

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[5] If evaluated in Euclidean signature the more singular behaviour arises because the replacement of the frequency integral by a Matsubara sum means one integrates only over spatial momenta d^3k .

[5] 若在欧几里得号差下计算，会出现更奇异的行为，这是因为将频率积分替换为松原求和后，我们仅需要对空间动量 d^3k 积分。

[6] Although controlled resummation can be possible for scalars whose mass vanishes at zero temperature, it need not be true in general that the perturbative breakdown associated with IR divergences can always be removed by resumming specific subsets of higher-order graphs. An example where this does not work arises when the total temperature-dependent mass $m^2 = m_0^2 + \delta m_T^2$ vanishes for some nonzero temperature. This corresponds to arranging the theory to sit at a critical point, for which it is well-known that mean-field (perturbative) calculations are simply not a good approximation.

[6] 尽管对零温下质量为零的标量，可以实现可控重求和，但一般来说，与红外发散相关的微扰失效并不总能通过重求和高阶图的特定子集来消除。当依赖温度的总质量 $m^2 = m_0^2 + \delta m_T^2$ 在某非零温度下等于零时，就会出现一个这种方法失效的例子。这对应理论恰好处于临界点，众所周知，平均场 (微扰) 计算在此处根本不是好的近似。

To make the transition to path integrals we start with the standard expression for transition amplitudes $\langle\varphi_1|U(t,t_0)|\varphi_2\rangle$, where $U(t,t_0)$ is the unitary time evolution operator defined in (10). These have the standard path-integral representation

为了过渡到路径积分，我们从跃迁振幅的标准表达式 $\langle\varphi_1|U(t,t_0)|\varphi_2\rangle$ 开始，其中 $U(t,t_0)$ 是式 (10) 定义的幺正时间演化算符。这些振幅有标准的路径积分表示

$$\langle\varphi_2|U(t,t_0)|\varphi_1\rangle = \int_{\varphi_1}^{\varphi_2}\mathcal{D}\phi\, e^{iS[\phi]}, \tag{59}$$

where $S[\phi]$ is the system’s classical action

其中 $S[\phi]$ 是系统的经典作用量

$$S[\phi] = \int_{t_0}^{t} \mathrm{d}t' L[\phi; t'], \tag{60}$$

and $|\varphi_j\rangle$ are eigenstates of the Schrödinger-picture field operator $\widehat{\phi}(\mathbf{x})$, so

且 $|\varphi_j\rangle$ 是薛定谔绘景场算符 $\widehat{\phi}(\mathbf{x})$ 的本征态，因此

$$\widehat{\phi}(\mathbf{x})|\varphi_j\rangle = \varphi_j(\mathbf{x})|\varphi_j\rangle, \tag{61}$$

and for notational clarity the x-dependence of the eigenvalues in Eq. (59) is suppressed when used as a label for a bra and ket. The limits of integration indicate that the integral sums over all configurations whose end points are chosen to be the specified initial and final eigenvalues: $\phi(\mathbf{x}, t_0) = \varphi_1(\mathbf{x})$ and $\phi(\mathbf{x}, t) = \varphi_2(\mathbf{x})$. The path integral (59) describes the probability amplitude that the system finds itself in the eigenstate $|\varphi_2\rangle$ at time t given it began in the eigenstate $|\varphi_1\rangle$ at the initial time t_0 .

且为了符号简洁，式 (59) 中本征值对 x 的依赖在用作左右矢标签时被省略。积分限表示积分对所有构型求和，这些构型的端点被选定为给定的初态和末态本征值: $\phi(\mathbf{x}, t_0) = \varphi_1(\mathbf{x})$ 和 $\phi(\mathbf{x}, t) = \varphi_2(\mathbf{x})$ 。路径积分 (59) 描述了系统初始时刻 t_0 处于本征态 $|\varphi_1\rangle$，而后在时刻 t 处于本征态 $|\varphi_2\rangle$ 的概率幅。

This can be turned into a path-integral representation for the density matrix by noting that the density matrix evolves as $\hat{\rho}(t) = U(t, t_0)\hat{\rho}_0 U^*(t, t_0)$ - cf. Eq. (9) - in the Schrödinger picture. This ensures its matrix elements can be written as

我们可以将其转化为密度矩阵的路径积分表示，只需注意到在薛定谔绘景下，密度矩阵的演化满足 $\hat{\rho}(t) = U(t, t_0)\hat{\rho}_0 U^*(t, t_0)$ ——参见式 (9)——这保证它的矩阵元可以写为

$$\langle\varphi_2|\hat{\rho}(t)|\varphi_1\rangle = \langle\varphi_2|U(t, t_0)\hat{\rho}_0 U^*(t, t_0)|\varphi_1\rangle \tag{62}$$

$$= \sum_{\varphi_3,\varphi_4} \langle\varphi_2|U(t, t_0)|\varphi_4\rangle\langle\varphi_4|\hat{\rho}_0|\varphi_3\rangle\langle\varphi_1|U(t, t_0)|\varphi_3\rangle^*$$

which inserts two resolutions of the identity. Using (59) one finds

这一步插入了两个单位算符的分辨率，利用式 (59) 可得

$$\langle\varphi_2|\hat{\rho}(t)|\varphi_1\rangle = \sum_{\varphi_3,\varphi_4} \int_{\varphi_4}^{\varphi_2} \mathcal{D}\phi^+ \int_{\varphi_3}^{\varphi_1} \mathcal{D}\phi^- e^{iS[\phi^+] - iS[\phi^-]} \langle\varphi_4|\hat{\rho}_0|\varphi_3\rangle, \tag{63}$$

where the path integration takes place over two independent field variables, labelled ϕ^+ and ϕ^- , that satisfy the distinct boundary conditions $\phi^+(\mathbf{x}, t) = \varphi_2(\mathbf{x})$, $\phi^+(\mathbf{x}, t_0) = \varphi_4(\mathbf{x})$ and $\phi^-(\mathbf{x}, t) = \varphi_1(\mathbf{x})$, $\phi^-(\mathbf{x}, t_0) = \varphi_3(\mathbf{x})$.

其中路径积分对两个独立场变量进行，分别标记为 ϕ^+ 和 ϕ^- ，二者满足不同边界条件 $\phi^+(\mathbf{x}, t) = \varphi_2(\mathbf{x})$ 、$\phi^+(\mathbf{x}, t_0) = \varphi_4(\mathbf{x})$ 和 $\phi^-(\mathbf{x}, t) = \varphi_1(\mathbf{x})$, $\phi^-(\mathbf{x}, t_0) = \varphi_3(\mathbf{x})$ 。

Expressions such as (63) are the point of departure for the Schwinger-Keldysh formalism - or 'in-in' or 'closed-time path' formalism - used in section "Secular Growth for Thermal Fields" to calculate field-theoretic quantities like Eq. (57) for thermal correlators [20]. For a simple example of how this works, notice that one can use Eq. (63) to write equal-time correlation functions in terms of the diagonal components of $\hat{\rho}$,

形如 (63) 的表达式是施温格-凯尔迪什形式 (也称“in-in”或闭合时间路径形式) 的出发点，我们在“热场的长期增长”一节中使用该形式计算热关联函数的场论量 (如式 (57))[20]。举一个简单的工作原理示例: 不难发现我们可以利用式 (63)，将等时关联函数用 $\hat{\rho}$ 的对角分量写出,

$$\operatorname{Tr}\left[\phi_H(t,\mathbf{x})\,\phi_H(t,\mathbf{x}')\,\hat{\rho}_0\right] = \operatorname{Tr}\left[\hat{\phi}(\mathbf{x})\,\hat{\phi}(\mathbf{x}')\,\hat{\rho}(t)\right] = \sum_{\varphi}\varphi(\mathbf{x})\,\varphi(\mathbf{x}')\,\langle\varphi\,|\hat{\rho}(t)|\,\varphi\rangle$$

$$= \sum_{\varphi,\varphi_3,\varphi_4}\varphi(\mathbf{x})\,\varphi(\mathbf{x}')\int_{\varphi_4}^{\varphi}\mathcal{D}\phi^{+}\int_{\varphi_3}^{\varphi}\mathcal{D}\phi^{-}e^{iS[\phi^{+}]-iS[\phi^{-}]}\,\langle\varphi_4\,|\hat{\rho}_0|\,\varphi_3\rangle\,, \tag{64}$$

where Heisenberg picture operators are defined in terms of Schrödinger picture operators by

其中海森堡绘景下的算符由薛定谔绘景下的算符按下式定义

$$\phi_H(t,\mathbf{x}) = U^*(t,t_0)\,\hat{\phi}(\mathbf{x})\,U(t,t_0)\,. \tag{65}$$

In this formalism, one interprets path integrations such as the one in (64) as performed over a deformed time contour which starts at the initial time t_0 , propagates out to the later time t (where the boundary conditions in (64) are identified), and then backwards to the initial time t_0 , as depicted in Fig. 4. The field variable living on either branch of the 'closed-time path' are treated as independent variables (ϕ^+ and ϕ^-) as conveyed by earlier formulas.

在该形式中，我们将 (64) 中的路径积分理解为在变形时间围道上进行: 围道从初始时刻 t_0 出发，传播到较晚时刻 t (此处对应 (64) 中的边界条件)，随后再回到初始时刻 t_0 ，如图 4 所示。和前面公式表述的一致，“闭合时间路径”两个分支上的场变量被视为独立变量 (ϕ^+ 和 ϕ^-)。

The Schwinger-Keldysh framework is most useful when one only has information about the initial state (i.e. being ignorant of the 'out' state) and so is well-equipped for dealing with non-equilibrium time evolution (generally the case in cosmology). It is also useful for time-evolving mixed states (including thermal states7) since quantum averages can be expressed in terms of expectation values over the 'doubled' degrees of freedom - what is useful about this is that it allows one to use standard techniques of QFT (e.g. Feynman diagrams) in this more complicated setting.

施温格-凯尔迪什框架最适用于仅掌握初态信息 (即不了解“出”态) 的场景，因此非常适合处理非平衡时间演化 (这在宇宙学中是普遍情况)。它也适用于混合态 (包括热态 [7]) 的时间演化，因为量子平均可以表示为“加倍”自由度上的期望——这套方法的优势在于，它允许我们在这种更复杂的场景下使用量子场论的标准技术 (例如费曼图)。

For the present purposes, formula (63) becomes most interesting in an open systems setting when the action $S[\phi_A,\phi_B]$ is a function of system ϕ_A and environment ϕ_B degrees of freedom. Using the notation from earlier, this means that the components of the full density matrix (63) here become instead

就本文的研究目的而言，当作用量 $S[\phi_A,\phi_B]$ 是系统自由度 ϕ_A 和环境自由度 ϕ_B 的函数时，开系场景下的公式 (63) 变得尤为重要。沿用此前的记号，这意味着此处全密度矩阵 (63) 的分量可改写为

$$\langle a_2,b_2|\hat{\rho}(t)|a_1,b_1\rangle = \sum_{a_3,a_4,b_3,b_4}\int_{a_4}^{a_2}\mathcal{D}\phi_A^+\int_{b_4}^{b_2}\mathcal{D}\phi_B^+\int_{a_3}^{a_1}\mathcal{D}\phi_A^-\int_{b_3}^{b_1}\mathcal{D}\phi_B^-$$
$$\times e^{iS[\phi_A^+,\phi_B^+]-iS[\phi_A^-,\phi_B^-]}\langle a_4,b_4|\hat{\rho}_0|a_3,b_3\rangle, \tag{66}$$

where the lower (and upper) end points on the path integrals are fixed at time t_0 (and t) as in (63). When the system and environment interact through an action of the form

其中和 (63) 一样，路径积分的下端点和上端点分别固定在时刻 t_0 和时刻 t。当系统和环境通过如下形式的作用量相互作用时

$$S[\phi_A,\phi_B] = S_A[\phi_A] + S_B[\phi_B] + S_{\text{int}}[\phi_A,\phi_B] \tag{67}$$

---

[7] When dealing with thermal states at temperature $1/\beta$, one usually further deforms the contour in Fig. 4 to include a third piece that points in the imaginary time direction starting at time t_0 , where one identifies t_0 with $t_0 + i\beta$ - ultimately this is a manifestation of the Kubo-Martin-Schwinger (KMS) detailed-balance condition obeyed by thermal correlators [21,22].

[7] 当处理温度为 $1/\beta$ 的热态时，我们通常会进一步变形图 4 中的围道，新增一段沿虚时间方向、从时刻 t_0 出发的部分，此时我们将 t_0 等同于 $t_0+i\beta$ ——这本质上是热关联函数满足的久保-马丁-施温格 (KMS) 细致平衡条件的体现 [21,22]。

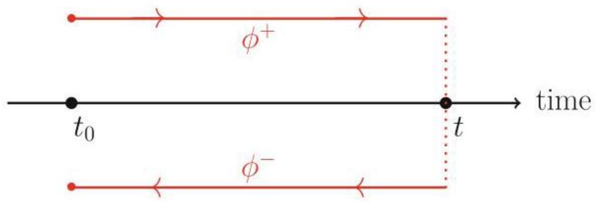

Fig. 4 Depiction of the closed-time path contour. Since only initial data is known (as opposed to the standard ”in-out” situation in scattering calculations), averages like Eq. (64) computed via path integrals start at time t_0 , flow out to t , and then back to t_0 . The value of the field on the upper and lower branches are denoted by ϕ^+ and ϕ^- and are treated as independent variables. In the literature, the contours are sometimes translated above and below the time axis by a tiny amount $\pm i\varepsilon$ to help with convergence of the path integrations

图 4 闭合时间路径围道示意图。由于仅已知初始数据 (和散射计算中标准的“in-out”情形不同)，通过路径积分计算的如式 (64) 的平均量从时刻 t_0 出发，延伸至 t ，再返回 t_0 。上下分支的场值分别记为 ϕ^+ 和 ϕ^- ，二者被处理为独立变量。文献中有时会将围道向时间轴上下平移极小量 $\pm i\varepsilon$ ，以促进路径积分收敛。

for some interaction S_{int} , then one can trace over the environment to find the elements of the (Schrödinger picture) reduced density matrix $\hat{\rho}_A$ to get the path-integral representation

对于某个相互作用 S_{int} ，我们可以对环境求迹得到 (薛定谔绘景下的) 约化密度矩阵 $\hat{\rho}_A$ 的元，从而得到路径积分表示

$$\langle a_2 |\hat{\rho}_A(t)| a_1\rangle = \sum_b \langle a_2, b |\hat{\rho}(t)| a_1, b\rangle \tag{68}$$

$$= \sum_{a_3,a_4} \int_{a_4}^{a_2} \mathcal{D}\phi_A^+ \int_{a_3}^{a_1} \mathcal{D}\phi_A^- e^{iS_A[\phi_A^+]-iS_A[\phi_A^-]+iS_{IF}[\phi_A^+,\phi_A^-]} \langle a_4 |\rho_A(t_0)| a_3\rangle .$$

This last expression packages the entire effect of the environment into $S_{IF}\left[\phi_A^+, \phi_A^-\right]$, called the influence functional, defined by

上述最后一个表达式将环境的全部效应归纳到 $S_{IF}\left[\phi_A^+, \phi_A^-\right]$ 中，$S_{IF}\left[\phi_A^+, \phi_A^-\right]$ 被称为影响泛函，其定义为

$$e^{iS_{IF}[\phi_A^+,\phi_A^-]} := \sum_{b,b_3,b_4} \int_{b_4}^{b} \mathcal{D}\phi_B^+ \int_{b_3}^{b} \mathcal{D}\phi_B^- \tag{69}$$

$$\times e^{iS_B[\phi_B^+]+iS_{\text{int}}[\phi_A^+,\phi_B^+]-iS_B[\phi_B^-]-iS_{\text{int}}[\phi_A^-,\phi_B^-]} \langle b_4 |\rho_B| b_3\rangle ,$$

which assumes an uncorrelated initial condition $\rho_0 = \rho_A(t_0) \otimes \rho_B$, as in (16).

它如式 (16) 一样，假设初始条件为无关联 $\rho_0 = \rho_A(t_0) \otimes \rho_B$

A few comments are in order. In general $S_{IF}\left[\phi_A^+, \phi_A^-\right]$ is composed of interactions between ϕ_A^+ and ϕ_A^- (as well as self-interactions for each). This is distinct from the usual situation in the Schwinger-Keldysh formalism - see Eq. (63) - where the actions for ϕ_A^+ and ϕ_A^- split apart. There are also unitary and non-unitary contributions to S_{IF} , and finally the interactions are generally non-local. [8] All these ingredients further drive home the point that effective descriptions for open systems can be very non-Wilsonian.

这里做一些说明: 一般情况下 $S_{IF}\left[\phi_A^+, \phi_A^-\right]$ 由 ϕ_A^+ 和 ϕ_A^- 之间的相互作用 (以及二者各自的自相互作用) 构成，这与施温格-凯尔迪什形式体系中的通常情况不同——见式 (63)，该体系中 ϕ_A^+ 和 ϕ_A^- 的作用量是分离的。此外，S_{IF} 存在幺正和非幺正贡献，且相互作用通常都是非定域的。[8] 所有这些性质都进一步印证了一个观点: 开放系统的有效描述完全可以是非威尔逊的。

In section "Hotspots and Influence Functionals" we explore in some detail how this works for a toy model of a black hole that has been devised to be solvable and yet also to capture important open-system features. We in particular use the influence functional to obtain an alternative derivation of a master equation and a stochastic Langevin equation.

在“热点与影响泛函”一节中，我们详细探讨了这套方法如何应用于一个可解的黑洞玩具模型，该模型恰好能够捕捉开放系统的核心特性。我们 specifically 利用影响泛函给出了主方程和随机朗之万方程的另一种推导。

Applications to Rindler Space

林德勒空间的应用

We next turn to some simple illustrative applications of these techniques in space-times with horizons. Applications of Open EFT techniques are still relatively recent for gravity and we try to choose examples that illustrate current developments.

我们接下来介绍这些技术在带视界时空中的几个简单说明性应用。开放 EFT 技术在引力中的应用目前仍属于较新的领域，我们选取的例子都旨在展示当前的研究进展。

We do so using applications to Rindler, de Sitter and simple black hole geometries in turn, starting in this sector with the simplest - Rindler - case. We begin in each case with simple qubit examples that behave very much like the thermal system described above and for which the simplicity of the qubit sector allows calculations to be very explicit and assumptions to be robustly tested. We then describe illustrative examples of secular growth in more fully field-theoretic systems.

我们将依次通过林德勒空间、德西特空间和简单黑洞几何这几个例子展开讨论，本节先从最简单的林德勒空间情况入手。每个案例我们都会先从简单量子比特示例开始，这些系统的行为和前文描述的热系统非常相近，而且量子比特部分结构简单，能够让计算过程非常清晰，假设也能得到可靠检验。随后我们会给出更完整的场论系统中长期增长的说明性示例。

Accelerated Qubit Thermalization

加速量子比特热化

We start by considering horizons generated by accelerated motion without a gravitational field. To explore this we use the same flat-space system as described above in section "Qubit Thermalization" - a qubit coupled to a massless scalar field - but with two important differences: the scalar field is prepared in its (Minkowski) vacuum $\rho_B = |\Omega\rangle\langle\Omega|$ (the $T \to 0$ limit of the above) and the qubit is uniformly accelerated rather than static.

我们首先考虑无引力场情况下由加速运动产生的视界。为研究这一问题，我们使用上文“量子比特热化”小节中介绍的同一平直时空系统——即一个耦合到无质量标量场的量子比特，但存在两处关键区别: 标量场处于其 (闵氏) 真空态 $\rho_B = |\Omega\rangle\langle\Omega|$ (上述系统的 $T \to 0$ 极限)，且量子比特做匀加速运动而非静止。

The qubit is a simple two-level accelerated DeWitt-Unruh detector [23, 24], whose evolution in perturbation theory is well-studied in the literature (at least for early times). We describe how these early treatments can be extended to give reliable predictions at the late times relevant to thermalization to the Unruh temperature (which lies beyond the domain of validity of the earlier perturbative studies).

该量子比特是一个简单的两能级加速狄维特-昂鲁探测器 [23,24]，其微扰论演化已有大量文献研究 (至少对早期时间演化而言)。我们说明如何拓展这些早期研究，以得到热化到昂鲁温度过程中晚期时间的可靠预测 (该区域超出了早期微扰研究的有效适用范围)。

The system describing the detector and the quantum field is again described by the unperturbed Hamiltonian $H_0 = H_A \otimes I_B + I_A \otimes H_B$, where H_B is precisely as given in (30) but with the qubit Hamiltonian generalized to include the time-dilation associated with its motion:

描述探测器和量子场的体系仍由未受扰哈密顿量 $H_0 = H_A \otimes I_B + I_A \otimes H_B$ 给出，其中 H_B 与式 (30) 给出的形式完全一致，但我们推广了量子比特哈密顿量，以纳入其运动带来的时间膨胀效应:

$$H_A = \mathbf{h}\frac{\mathrm{d}\tau}{\mathrm{d}t} \quad \text{with} \quad \mathbf{h} := \frac{\omega}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{70}$$

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[8] Nonlocality might be especially relevant in discussion of gravitation backgrounds with horizons, where other types of hypothetical non-local effects are sometimes considered.

[8] 非定域性在讨论带视界的引力背景时可能尤为重要，这类背景中有时也会讨论其他类型的假设非定域效应。

---

where τ is the proper time $\mathrm{d}\tau^2 = -\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$ evaluated along the qubit’s worldline $x^\mu = y^\mu(\tau)$. This means that $\omega > 0$ is the splitting between qubit energy levels as measured in the rest frame of the qubit. The complete Hamiltonian is $H_0 + H_{\text{int}}$ where the qubit interaction in Schrödinger picture also contains a time-dilation factor, with Hamiltonian

其中 τ 是沿量子比特世界线 $x^\mu = y^\mu(\tau)$ 计算得到的固有时 $\mathrm{d}\tau^2 = -\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$ 。这意味着 $\omega > 0$ 是量子比特静止系中测量得到的能级分裂。完整哈密顿量为 $H_0 + H_{\text{int}}$ ，薛定谔绘景下的量子比特相互作用也包含一个时间膨胀因子，哈密顿量为

$$H_{\text{int}} = g\widehat{\alpha} \otimes \widehat{\phi}[y(\tau)]\,\frac{\mathrm{d}\tau}{\mathrm{d}t} \quad \text{where} \quad \widehat{\alpha} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{71}$$

where the ’hat’ again denotes a Schrödinger-picture operator and the dimensionless coupling $g \ll 1$ is chosen small enough to justify perturbative methods.

其中带“帽子”的符号仍表示薛定谔绘景算符，无量纲耦合 $g \ll 1$ 取值足够小，保证微扰方法适用。

With these choices the free evolution is given by the time-ordered expression

在上述设定下，自由演化由如下时序表达式给出

$$U_0(t) = \mathcal{T}\exp\left(-i\int_0^t \mathrm{d}s H_0\right) = e^{-i\mathbf{h}\tau(t)} \otimes e^{-iH_B t} \tag{72}$$

and so the interaction-picture interaction Hamiltonian becomes

因此相互作用绘景下的相互作用哈密顿量为

$$V(t) = U_0^\dagger(t) H_{\text{int}} U_0(t) = g\phi[y(\tau)] \otimes \alpha(\tau) \frac{\mathrm{d}\tau}{\mathrm{d}t}, \tag{73}$$

where

其中

$$\alpha(\tau) = e^{i\mathbf{h}\tau} \widehat{\alpha} e^{-i\mathbf{h}\tau} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} e^{-i\omega\tau} + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} e^{i\omega\tau}. \tag{74}$$

From here on we proceed precisely as in section ”Qubit Thermalization”, so we report only the parts of the calculation that change. The first change is to choose the field to be prepared in the Minkowski vacuum $|\Omega\rangle$ rather than a thermal state, so

从这里开始我们的计算步骤与“量子比特热化”小节完全一致，因此我们仅给出发生变化的计算部分。第一处变化是我们将场设定为处于闵氏真空 $|\Omega\rangle$ 而非热态，因此

$$\rho_B = |\Omega\rangle\langle\Omega|. \tag{75}$$

This is also the $T \to 0$ limit of the state considered in section ”Qubit Thermalization” and so for a static qubit situated at $\mathbf{x} = \mathbf{x}_0$ the Wightman function can be found by taking $\beta \to \infty$ in (34):

这也是“量子比特热化”小节中讨论的态的 $T \to 0$ 极限，因此对位于 $\mathbf{x} = \mathbf{x}_0$ 的静止量子比特，可以通过对式 (34) 取 $\beta \to \infty$ 得到怀特曼函数:

$$W(s) = \langle \Omega | \phi(\mathbf{x}_0, s) \phi(\mathbf{x}_0, 0) | \Omega \rangle = -\frac{1}{4\pi^2 (s - i\varepsilon)^2}, \tag{76}$$

implying $R(\omega) = 0$ - cf. Eq. (38). Unsurprisingly, stationary qubits that are initially in their ground state remain there despite coupling to the field if the field is prepared in its own ground state.

由此可得 $R(\omega) = 0$ ——参见式 (38)。如果场本身处于基态，那么初始处于基态的静止量子比特即使与场耦合，也会保持在基态，这一结果并不意外。

We instead move the qubit along a uniformly accelerated trajectory $x^\mu = y^\mu(\tau)$

我们转而让量子比特沿匀加速轨迹 $x^\mu = y^\mu(\tau)$ 运动

$$\text{with } y^\mu(\tau) = \left[\frac{1}{a}\sinh(a\tau), \frac{1}{a}\cosh(a\tau), 0, 0\right], \tag{77}$$

where $a > 0$ denotes the qubit’s proper acceleration and τ in this parameterization denotes proper time along the curve as measured using the Minkowski metric. A scalar field’s Wightman function evaluated along this worldline is evaluated in closed-form in [27, 28] , and the massless limit of this result is [9]

其中 $a > 0$ 表示量子比特的固有加速度，该参数化中 τ 表示用闵氏度量得到的曲线上的固有时。标量场沿该世界线计算得到的怀特曼函数在 [27, 28] 中已经有闭式结果，其无质量极限为 [9]

$$W(\tau) = \langle \Omega | \phi[y(\tau)] \phi[y(0)] | \Omega \rangle = -\frac{a^2}{16\pi^2 [\sinh(a\tau/2) - i\varepsilon]^2}, \tag{78}$$

which has the thermal form - compare with (34) - with $\beta = 2\pi/a$ corresponding to the usual Unruh temperature.

该函数具有热形式——与式 (34) 对比——其中 $\beta = 2\pi/a$ 对应常规昂鲁温度。

From here on the calculation follows along very much the same lines as in section ”Qubit Thermalization”. For qubits initially in their ground state and uncorrelated with the field, the leading rest-frame perturbative excitation rate agrees with earlier predictions [23-25]:

自此往下，计算过程与“量子比特热化”一节的思路大体一致。对于初始处于基态且与场无关联的量子比特，其静止系领头阶微扰激发率与先前的预测 [23-25] 一致:

$$\frac{\partial \rho_{\uparrow\uparrow}}{\partial \tau} \simeq g^2 R(\omega) \quad \text{where} \quad R(\omega) = \frac{\omega}{2\pi} \frac{1}{e^{2\pi\omega/a} - 1}, \tag{79}$$

for proper times $2\pi/a \ll \tau \ll 2\pi/\left(g^2 a\right)$. Although this perturbative result breaks down at large times, the arguments of section ”Qubit Thermalization” show that evolution is reliably well-approximated by a Markovian process within the parameter regime (45), which in this instance can be written

对于原时 $2\pi/a \ll \tau \ll 2\pi/\left(g^2 a\right)$。尽管该微扰结果在大时间下失效，但“量子比特热化”一节的论证表明，在参数范围 (45) 内，演化可以用马尔可夫过程很好地近似，该情形下参数范围可写为

$$1 \gg \frac{2\pi\omega}{a} \gg \frac{g^2}{2\pi}. \tag{80}$$

In this regime the proper-time evolution is given by

在此范围内，原时演化由下式给出

$$\frac{\partial \rho_{\uparrow\uparrow}}{\partial \tau} \simeq g^2 R(\omega) - 2g^2 C(\omega) \rho_{\uparrow\uparrow}(\tau), \tag{81}$$

with

其中

$$C(\omega) = \frac{\omega}{4\pi} \coth\left(\frac{\pi\omega}{a}\right). \tag{82}$$

[9] It can be tempting to rewrite $\sinh(a\tau/2) - i\varepsilon$ as $\sinh[(a(\tau - i\varepsilon)/2]$ with the reasoning that these are equivalent because infinitesimal $\varepsilon > 0$ is important only near $\tau = 0$ [27]. Although this reasoning is not false for real τ , this replacement can be dangerous where τ is not real because it does not preserve important properties like the KMS condition mentioned in Footnote 7.

[9] 人们很容易将 $\sinh(a\tau/2) - i\varepsilon$ 改写为 $\sinh[(a(\tau - i\varepsilon)/2]$ ，理由是二者等价，因为只有在 $\tau = 0$ 附近无穷小 $\varepsilon > 0$ 才重要 [27]。尽管对于实 τ 来说这一推导并没有错，但当 τ 不是实数时，这种替换存在风险——因为它不保留重要性质，例如脚注 7 中提到的 KMS 条件。

The late-time solutions are given by (48) and (53) and describe thermalization to the Unruh temperature with relaxation times in the qubit rest frame given by

晚时解由式 (48) 和 (53) 给出，描述了系统热化至盎鲁温度的过程，量子比特静止系中的弛豫时间由下式给出

$$\xi_c = 2\xi_d = \frac{1}{g^2 C(\omega)} = \frac{4\pi}{g^2\omega} \tanh\left(\frac{\pi\omega}{a}\right) \simeq \frac{4\pi^2}{g^2 a}\left\{1 + O\left[\left(\frac{\pi\omega}{a}\right)^2\right]\right\}. \tag{83}$$

Secular Growth and the Minkowski Vacuum

长期增长与闵可夫斯基真空

We next turn to a more fully field-theoretic example for which the interaction involves only quantum fields. In particular we show how loop corrections involving a scalar field prepared in its interacting vacuum in the presence of a self-interaction $\frac{1}{4!}\lambda\phi^4$ can in some circumstances have the same kinds of secular growth [10] in its propagators as found above for the self-interacting thermal case. In doing so we will resolve a puzzle: if the Minkowski vacuum can describe thermal physics for some observers, then why does not bog-standard zero-temperature perturbation theory (with the field prepared in its Minkowski vacuum) also give rise to secular growth effects and late-time perturbative breakdown?

我们接下来转向一个更完整的量子场论例子，其中相互作用仅涉及量子场。我们特别展示，对于自相互作用 $\frac{1}{4!}\lambda\phi^4$ 存在时处于相互作用真空的标量场，圈修正在某些情况下可以在传播子中产生与之前自相互作用热情形相同类型的长期增长 [10] 。在此过程中我们将解决一个谜题: 如果闵可夫斯基真空对部分观测者而言可以描述热物理，那么为什么标准的零温微扰论 (场处于闵可夫斯基真空) 也会产生长期增长效应和晚期微扰失效?

To understand why, we re-evaluate the graph of Fig. 2 at zero temperature. For these purposes it is useful to recall that the lowest-order position-space propagator at zero temperature is (for nonzero scalar mass)

为理解其中原因，我们在零温下重新计算图 2 的图。为此我们首先回顾，零温下最低阶位置空间传播子 (对于非零标量质量) 为

$$G_0(x;y) = \frac{1}{4\pi^2}\frac{m}{\sqrt{(x-y)^2+i\varepsilon}}K_1\left[m\sqrt{(x-y)^2+i\varepsilon}\right], \tag{84}$$

where $(x-y)^2 = \eta_{\mu\nu}(x-y)^\mu(x-y)^\nu$ is negative for time-like separations and positive for space-like separations and $K_v(z)$ is a modified Bessel function of the second kind. The asymptotic form of $K_v(z)$ reproduces the usual massless limit:

其中对于类空间隔 $(x-y)^2 = \eta_{\mu\nu}(x-y)^\mu(x-y)^\nu$ 为正，类时间隔 $(x-y)^2 = \eta_{\mu\nu}(x-y)^\mu(x-y)^\nu$ 为负，$K_v(z)$ 是第二类修正贝塞尔函数。$K_v(z)$ 的渐近形式给出了常规无质量极限:

$$G_0(x;y) = \frac{1}{4\pi^2}\frac{1}{(x-y)^2+i\varepsilon}\quad (m=0). \tag{85}$$

For a massive scalar Fig. 2 evaluates to give

对于有质量标量，计算图 2 可得

$$G_{\text{tad}}(x;y) = -i\sum(0)\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{e^{ip\cdot(x-y)}}{(p^2+m^2-i\varepsilon)^2} = -\frac{\delta m^2}{8\pi^2}K_0\left[m\sqrt{(x-y)^2+i\varepsilon}\right],$$

(86)

where the zero-momentum self-energy is

其中零动量自能为

$$\sum(0) = -\delta m_{\text{ct}}^2 + 3i\lambda\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2+m^2-i\varepsilon}, \tag{87}$$

with δm_{ct}^2 the mass counter-term that subtracts the UV-divergent part of the integral and $\delta m^2 = -\sum(0)$ is the UV finite mass shift after renormalization.

δm_{ct}^2 是抵消积分紫外发散部分的质量抵消项，$\delta m^2 = -\sum(0)$ 是重整化后的紫外有限质量移动。

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[10] See also [26].

[10] 另见文献 [26]。

In the massless limit this evaluates to

在无质量极限下计算可得

$$G_{\text{tad}}(x;y) = \frac{\delta m^2}{8\pi^2}\ln\left[\mu\sqrt{(x-y)^2+i\varepsilon}\right]\ (\text{ when } m=0) \tag{88}$$

up to a spacetime-independent IR-divergent constant. Here μ is the renormalization scale for the renormalized coupling $\lambda(\mu)$, whose precise value depends on how this IR divergence is regulated but plays no role when tracking the dependence of the result on $x-y$.

结果差一个不依赖时空的红外发散常数。此处 μ 是重整化耦合 $\lambda(\mu)$ 的重整化标度，其具体取值依赖于该红外发散的调节方式，但在追踪结果对 $x-y$ 的依赖关系时不起作用。

We now evaluate the propagator using coordinates adapted to accelerating observers, for which the flat metric becomes

我们现在使用适配加速观测者的坐标计算传播子，在该坐标下平直度规变为

$$\mathrm{d}s^2 = \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -(a\xi)^2\,\mathrm{d}\tau^2 + \mathrm{d}\xi^2 + \mathrm{d}y^2 + \mathrm{d}z^2. \tag{89}$$

Imagine now choosing both x^μ and y^μ to both lie along the specific accelerating worldline described by

现在想象令 x^μ 和 y^μ 都落在由下式描述的特定加速世界线上

$$x = \xi\cosh(a\tau) \quad \text{and} \quad t = \xi\sinh(a\tau), \tag{90}$$

with the other two coordinates (y and z) fixed. We evaluate $G(x,y)$ with $x^2 = y^2$ and $x^3 = y^3$ and choose a particular Rindler observer - i.e. fixed ξ - on whose accelerating worldline both x^μ and y^μ lie (and so are separated purely by a shift in Rindler time, τ). We choose in particular the specific trajectory $\xi = 1/a$, for which Rindler time is also the proper time along the curve and the proper acceleration is a .

另外两个坐标 (y and z) 固定。我们代入 $x^2 = y^2$ 和 $x^3 = y^3$ 计算 $G(x,y)$ ，选取一个特定的林德勒观测者——即固定 ξ ，x^μ 和 y^μ 都落在该观测者的加速世界线上 (因此两点仅相差林德勒时间 τ 的平移)。我们特别选取轨迹 $\xi = 1/a$ ，对于该轨迹，林德勒时间同时也是世界线的固有时，固有加速度为 a 。

The invariant separation between two points separated by proper time τ is then given by

那么间隔固有时 τ 的两点之间的不变间隔为

$$-(x-y)^2 = \frac{4}{a^2}\sinh^2\left(\frac{a\tau}{2}\right) \simeq \frac{1}{a^2}e^{a\tau}\left[1 + O\left(e^{-2a\tau}\right)\right], \tag{91}$$

where the final approximate equality gives the asymptotic form when $a\tau \gg 1$. For such an observer (with $a\tau \gg 1$) Eq. (88) implies an asymptotic time-dependence of $G_{\text{tad}}(\tau)$ of the form

最后的近似等式给出 $a\tau \gg 1$ 时的渐近形式。对于这类观测者 (满足 $a\tau \gg 1$)，式 (88) 表明 $G_{\text{tad}}(\tau)$ 的渐近时间依赖形式为

$$G_{\text{tad}}(\tau) = \frac{\delta m^2}{8\pi^2}\ln\left[\mu\sqrt{(x-y)^2 + i\varepsilon}\right] \simeq \frac{\delta m^2 a\tau}{16\pi^2} + \text{ subdominant}\,. \tag{92}$$

What value should be chosen for δm^2 in this last expression? As discussed earlier, the tadpole loop diverges in the UV and this divergence is cancelled by the mass counter-term, leaving a finite residual whose value depends on the renormalization scheme. A Minkowski observer would effectively choose $\delta m_M^2 = -\sum_M(0) = 0$ so that the unperturbed mass parameter is the physical mass. A punishment for not doing so would be to have IR divergences appear even at zero temperature in graphs like Fig. 3. A Rindler observer would instead choose a counter-term that ensures the sum of the counter-term and tadpole graph vanishes if evaluated in the Rindler ground state [11] rather than the Minkowski one, for similar reasons.

在最后这个表达式中，δm^2 应当取何值？正如前文讨论，蝌蚪图在紫外区发散，该发散可由质量抵消项消除，仅留下依赖于重整化方案的有限残差。闵氏观者会自然选取 $\delta m_M^2 = -\sum_M(0) = 0$，使得未受扰质量参数等于物理质量。不这么做的后果是，即便在零温下，类似图 3 的费曼图也会出现红外发散。出于类似原因，林德勒观者则会选择满足如下条件的抵消项: 当在林德勒基态 [11] 而非闵氏基态中计算时，抵消项与蝌蚪图的和为零。

The Minkowski observer's choice sets $\delta m^2 = 0$ and so (92) gives zero. The Rinder observer's choice instead no longer completely cancels the tadpole graph when it is evaluated in the Minkowski vacuum and so differs from the Minkowski choice by a finite amount. A standard calculation using the Rindler vacuum gives $[27, 29-33]$

闵氏观者的选择固定了 $\delta m^2 = 0$，因此 (92) 式给出结果为零。而林德勒观者的选择在闵氏真空中计算蝌蚪图时，无法完全抵消该图，因此与闵氏观者的选择相差一个有限量。利用林德勒真空进行标准计算可得 $[27, 29-33]$

$$\delta m_a^2 = \frac{\lambda a^2}{16\pi^2}. \tag{93}$$

Using this in (92) then gives

将其代入 (92) 式后可得

$$G_{\rm tad}(\tau) = \frac{\lambda a^3 \tau}{(16\pi^2)^2} + \text{ subdominant}, \tag{94}$$

which precisely agrees with the thermal result (57) provided we identify temperature with acceleration in the usual way: $T = a/2\pi$.

只要我们按照常规方式将温度等价为加速度: $T = a/2\pi$，该结果就与热结果 (57) 完全一致。

If the Minkowski observer's counter-term choice had instead been made then secular growth would have instead appeared in the Rindler correlation function. Said differently, secular growth at late Rindler time cannot be avoided for both the Minkowski and Rindler vacua, and once it is excluded from one vacuum it necessarily appears for the other and does so in precisely the way that would have been expected for a thermal state.

如果采用闵氏观者的抵消项选择，那么林德勒关联函数中就会出现长期增长。换句话说，林德勒晚期的长期增长不可能同时在闵氏真空和林德勒真空中被消除: 它被从一个真空中排除后，必然会出现在另一个真空中，而且其出现方式完全符合热状态的预期。

Applications to de Sitter Space

德西特空间的应用

We next turn to the simplest curved-space examples, which involve the de Sitter and near-de Sitter cosmologies likely to be associated with Dark-Energy dominated late-time cosmologies or inflationary cosmologies at very early times. de Sitter geometries are simple in the sense that de Sitter space shares the same number of isometries as does flat space, despite the presence of curvature. Because of this symmetry more explicit calculations are known for these geometries than for less symmetric ones that share the existence of horizons (like black holes).

我们接下来讨论最简单的弯曲空间例子，这类例子涉及德西特和近德西特宇宙学，它们很可能与暗能量主导的晚期宇宙，或是极早期的暴胀宇宙相关。德西特几何的简单之处在于: 尽管存在曲率，德西特空间拥有和平直空间一样多的等距对称性。得益于这种对称性，相比其他同样存在视界但对称性更低的几何 (比如黑洞)，我们对德西特几何能进行更多更明确的计算。

The geometry of interest is a spatially flat, homogeneous, and isotropic metric of the Friedmann-LeMaitre-Robertson-Walker (FLRW) form

我们所研究的几何是空间平坦、均匀且各向同性的弗里德曼-勒梅特-罗伯逊-沃克 (FLRW) 度规

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\mathbf{x}\cdot\mathrm{d}\mathbf{x} = a^2(\eta)\left[-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}\cdot\mathrm{d}\mathbf{x}\right], \tag{95}$$

where the geometry is specified in terms of a time-dependent scale factor $a(t)$, and for any given $a(t)$ conformal time η and cosmic time t are related by $\mathrm{d}t = a\,\mathrm{d}\eta$. The nonzero components of the Riemann tensor for this geometry are

其中几何由依赖时间的标度因子 $a(t)$ 描述，对任意给定的 $a(t)$ ，共形时间 η 和宇宙时间 t 满足关系 $\mathrm{d}t = a\,\mathrm{d}\eta$ 。该几何里黎曼张量的非零分量为

$$R^0{}_{i0j} = -qH^2 g_{ij} \quad \text{and} \quad R^i{}_{jkl} = H^2\left(\delta^i_k g_{jl} - \delta^i_l g_{jk}\right) \tag{96}$$

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[11] The Rindler state is the ground state of the Rindler Hamiltonian, defined as the Poincaré boost generator that generates translations in Rindler time.

[11] 林德勒态是林德勒哈密顿量的基态，被定义为生成林德勒时间平移的庞加莱 boost 生成元。

(plus those related to these by permuting indices), where

(以及那些通过指标置换得到的分量)，其中

$$H(t) := \frac{\dot{a}}{a} \text{ and } q(t) := -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \varepsilon_1 \text{ where } \varepsilon_1(t) := -\frac{\dot{H}}{H^2} \tag{97}$$

and overdots (primes) denote $\mathrm{d}/\mathrm{d}t$ ($\mathrm{d}/\mathrm{d}\eta$) . de Sitter space is the special case

点号 (撇号) 表示 $\mathrm{d}/\mathrm{d}t$ ($\mathrm{d}/\mathrm{d}\eta$) 。德西特空间是如下特殊情况

$$a = e^{Ht} = -\frac{1}{H\eta} \text{ where } H \text{ is constant (and so } q = -1 \text{ and } \varepsilon_1 = 0 \text{)}, \tag{98}$$

and for this specific choice the Riemann tensor has the maximally symmetric form $R^{\mu}{}_{\upsilon\lambda\rho} = H^2\left(\delta^{\mu}_{\lambda}g_{\upsilon\rho} - \delta^{\mu}_{\rho}g_{\upsilon\lambda}\right)$.

在这个特殊选择下，黎曼张量具有最大对称形式 $R^{\mu}{}_{\upsilon\lambda\rho} = H^2\left(\delta^{\mu}_{\lambda}g_{\upsilon\rho} - \delta^{\mu}_{\rho}g_{\upsilon\lambda}\right)$.

The quantization of such systems is done semiclassically, with all fields (including the metric) split into a classical background plus a quantum fluctuation. This makes sense if we work within the spirit of effective field theories for gravity (GREFT), since these allow one to systematically ask when and why such semiclassical methods are justified. See the accompanying chapters in this review for more about these techniques (and see [34,35]).

这类系统的量子化采用半经典方法: 所有场 (包括度规) 都分解为经典背景加上量子涨落。这一处理符合引力有效场论 (GREFT) 的思路，因为有效场论可以让我们系统地分析半经典方法在什么条件下、为什么是合理的。关于这些方法的更多内容参见本综述的对应章节 (亦可参见文献 [34,35])。

A massless scalar field that only couples minimally to gravity within an FLRW universe satisfies

FLRW 宇宙中，仅与引力最小耦合的无质量标量场满足

$$\begin{aligned} -\Box\phi &= -\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\upsilon}\partial_\upsilon\phi\right) = \ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi \\ &= \frac{1}{a^2}\left(\phi'' + \frac{2a'}{a}\phi' - \nabla^2\phi\right) = 0, \end{aligned} \tag{99}$$

where $\nabla^2 = \delta^{ij}\partial_i\partial_j$. Particle states for such a field can be labelled using momenta k because the spatial slices of the geometry are flat (and so translation invariant).

其中 $\nabla^2 = \delta^{ij}\partial_i\partial_j$ 。因为该几何的空间切片是平坦的 (因此具有平移不变性)，这类场的粒子态可以用动量 k 标记。

Specializing to de Sitter space and expanding the field in terms of the corresponding creation and annihilation operators

specialize 到德西特空间，并将场用相应的产生湮灭算符展开

$$\phi(x) = \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\left[\upsilon_{\mathbf{k}}(x)\,\mathfrak{a}_{\mathbf{k}} + \upsilon^*_{\mathbf{k}}(x)\,\mathfrak{a}^*_{\mathbf{k}}\right] \text{ with } \upsilon_{\mathbf{k}}(x) = \frac{u_{\mathbf{k}}(\eta)}{a}e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{100}$$

implies

可得

$$u''_{\mathbf{k}} + \left(\mathbf{k}^2 - \frac{2}{\eta^2}\right) u_{\mathbf{k}} = 0, \tag{101}$$

for which the normalized solutions are

其归一化解为

$$u_{\mathbf{k}}(\eta) = \frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta} \tag{102}$$

$$\text{and so } v_{\mathbf{k}}(a, \mathbf{x}) = \frac{1}{\sqrt{2k^3}} \left(\frac{k}{a} + iH\right) e^{i[(k/aH) + \mathbf{k}\cdot\mathbf{x}]}$$

(with $k := |\mathbf{k}|$) ensuring the standard commutation relations $\left[\mathfrak{a}_{\mathbf{k}}, \mathfrak{a}^*_{\mathbf{q}}\right] = \delta^3(\mathbf{k} - \mathbf{q})$. The vacuum state $|\Omega\rangle$ defined by $\mathfrak{a}_{\mathbf{k}}|\,\Omega\rangle = 0$ is called the Bunch-Davies vacuum.

(其中 $k := |\mathbf{k}|$) 保证了标准对易关系 $\left[\mathfrak{a}_{\mathbf{k}}, \mathfrak{a}^*_{\mathbf{q}}\right] = \delta^3(\mathbf{k} - \mathbf{q})$ 。这个真空态 $|\Omega\rangle$ defined by $\mathfrak{a}_{\mathbf{k}}|\,\Omega\rangle = 0$ 被称为邦奇-戴维斯真空。

For these modes physical momenta $\mathbf{p}(t) = \mathbf{k}/a(t)$ are time-dependent and fall monotonically and so for every mode there is a time t_{he} after which $|\mathbf{p}(t)| < H$, with crossover between these regimes ('horizon exit') occurring when $aH = k$. (Equivalently, the sweep from $-\infty < t < \infty$ corresponds to $-\infty < \eta < 0$ with $\eta \to 0$ in the far future and horizon exit occurring when $k\eta = -1$.) The modes (102) are oscillatory in the remote past (when $k\eta \ll -1$) and are chosen to resemble standard flat-space modes in this regime. Their motion stops being adiabatic after horizon exit, with (102) showing they stop oscillating (or 'freeze') once $|k\eta| \ll 1$.

对于这些模式，物理动量 $\mathbf{p}(t) = \mathbf{k}/a(t)$ 随时间变化且单调递减，因此对任意模式都存在时刻 t_{he} ，在该时刻之后满足 $|\mathbf{p}(t)| < H$ ，两个区域之间的交叉（“视界出射”）发生在 $aH = k$ 时。(等价地，从 $-\infty < t < \infty$ 的演化对应 $-\infty < \eta < 0$ ，远未来满足 $\eta \to 0$ ，视界出射发生在 $k\eta = -1$ 时。) 模式 (102) 在极早期 (当 $k\eta \ll -1$ 时) 呈振荡形式，且被设定为在该区域与标准平直空间模式一致。视界出射后，它们的运动不再满足绝热性，(102) 表明一旦 $|k\eta| \ll 1$ ，它们就会停止振荡 (或称“冻结”)。

Explicit expressions for massive mode functions are also known for de Sitter geometries, though we do not need them in what follows. One result we do use however is the expression (for a massive scalar field) for the renormalized expectation $\langle\phi^2(x)\rangle = \langle\Omega\,|\phi^2(x)|\,\Omega\rangle$ using the Bunch-Davies vacuum in de Sitter space. This is an ultraviolet divergent quantity and is independent of x for massive fields. Renormalizing so that it vanishes when $H \to 0$ leaves a finite and nonzero value for de Sitter given by

德西特几何中也已知有质量模式函数的显式表达式，不过我们后续推导不需要用到它们。但我们会用到一个结论: 在德西特空间中采用邦奇-戴维斯真空时，有质量标量场重整化期望 $\langle\phi^2(x)\rangle = \langle\Omega\,|\phi^2(x)|\,\Omega\rangle$ 的表达式。这是一个紫外发散量，且对于有质量场，它与 x 无关。做重整化使得该量在 $H \to 0$ 时为零后，德西特空间会留下一个非零有限值，由下式给出

$$\langle \phi^2(x) \rangle = \frac{3H^4}{8\pi^2 m^2}. \tag{103}$$

The divergence of this result as $m \to 0$ reflects the IR divergence that arises in this limit, as can be seen directly using the massless mode functions (102):

该结果当 $m \to 0$ 时发散, 反映了此极限下出现的红外发散, 这一点可以直接用无质量模式函数 (102) 看出:

$$\langle \phi^2(x) \rangle_{\text{massless}} = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} |v_{\mathbf{k}}(x)|^2 = \frac{1}{2\pi^2} \int_0^\infty \frac{\mathrm{d}k}{k} \left(k^3 |v_{\mathbf{k}}(x)|^2 \right), \tag{104}$$

where the second equality uses that $|v_{\mathbf{k}}(x)|^2$ is independent of the direction of $\mathbf{k}$. This expression diverges as $k \to 0$ due to the small- k behaviour of $|v_{\mathbf{k}}|^2$ seen in (102). For later purposes we remark that if this integral is regulated in the UV by multiplying by a window function $\xi_\Lambda [k/a(t)]$ that discriminates against UV momenta - such as if $\xi_\Lambda(z) = 1$ for $z \ll \Lambda$ and $\xi_\Lambda(z) = 0$ for $z \gg \Lambda$ - then

其中第二个等式利用了 $|v_{\mathbf{k}}(x)|^2$ 与 $\mathbf{k}$ 的方向无关。该表达式当 $k \to 0$ 时发散，这源于 (102) 中 $|v_{\mathbf{k}}|^2$ 的小 k 行为。为了后续方便，我们说明: 如果该积分在紫外通过乘窗口函数 $\xi_\Lambda [k/a(t)]$ 来正则化，该窗口函数会过滤掉紫外动量——例如当 $z \ll \Lambda$ 满足 $\xi_\Lambda(z) = 1$ 、当 $z \gg \Lambda$ 满足 $\xi_\Lambda(z) = 0$ ——那么

$$\begin{aligned} \partial_t \langle \phi^2(x) \rangle_{\text{massless}} &= Ha\partial_a \int \frac{\mathrm{d}^3 k}{(2\pi)^3} |v_{\mathbf{k}}(x)|^2 \xi_A(k/a) \\ &= -\frac{H}{2\pi^2} \int_0^\infty \mathrm{d}k \frac{\partial}{\partial k} \left(k^3 |v_{\mathbf{k}}(x)|^2 \xi_A \right) = \frac{H^3}{4\pi^2}, \end{aligned} \tag{105}$$

and so is infrared finite. This uses that $a\partial_a = -k\partial_k$ when acting on any function of k/a together with (102) and the properties $\xi_\Lambda(\infty) = 0$ and $\xi_\Lambda(0) = 1$. Having $\langle \phi^2 \rangle$ be linear in t makes many of its implications resemble those of a random walk.

因此它是红外有限的。推导利用了 $a\partial_a = -k\partial_k$ 作用在任意 k/a 的函数上的性质，结合 (102) 以及性质 $\xi_\Lambda(\infty) = 0$ 和 $\xi_\Lambda(0) = 1$ 。当 $\langle \phi^2 \rangle$ 对 t 呈线性时，它的很多推论都与随机游走的推论相似。

The remainder of this section briefly sketches several applications of Open EFTs to de Sitter geometries. As in the previous section we first examine the simplest case of a qubit interacting with a quantum field and then discuss several more field-theoretic situations both of which involve the need to resum secular growth.

本节剩余部分简要概述开敞有效场论 (Open EFT) 在德西特几何中的若干应用。和上一节一样，我们首先考察量子比特与量子场相互作用的最简单情形，随后讨论另外两种场论场景，这两种场景都需要对久期增长进行重求和。

Qubit Thermalization

量子比特热化

One can probe the structure of de Sitter space by again considering a qubit coupled to a scalar field along the lines of section ”Accelerated Qubit Thermalization”. The presence of an event horizon - in this case the de Sitter horizon caused by the ever-expanding nature of the universe - gives rise to the so-called Gibbons-Hawking temperature $T_{GH} = H/(2\pi)$. The qubit again thermalizes to this temperature in much the same way as in earlier sections, but this example also shows that horizons can capture other features also present in thermal baths: in this case the phenomenon of ’critical slowing down’ in which thermalization becomes very slow when the effective mass of the scalar is tuned to be very small.

我们可以沿用“加速量子比特热化”一节的思路，再次通过耦合标量场的量子比特探测德西特空间的结构。事件视界——此处是由宇宙持续膨胀产生的德西特视界——会产生所谓的吉本斯-霍金温度 $T_{GH} = H/(2\pi)$ 。量子比特会和前文情形一样热化到该温度，但这个例子也表明，视界可以具备热浴也拥有的其他特征: 即“临界慢化”现象，当标量场的有效质量被调至非常小时，热化过程会变得极慢。

For later convenience we set up the problem in a way that is easy to generalize to other (e.g. black hole) geometries. To this end we write metric in a slightly more general form

为方便后续推导，我们以易于推广到其他 (例如黑洞) 几何的方式构建问题，为此我们将度规写为更一般的形式

$$\mathrm{d}s^2 = -f(\mathbf{x})\,\mathrm{d}t^2 + \gamma_{ij}(\mathbf{x},t)\,\mathrm{d}x^i\,\mathrm{d}x^j, \tag{106}$$

for which de Sitter space corresponds to the choices $f(\mathbf{x}) = 1$ and $\gamma_{ij} = e^{2Ht}\delta_{ij}$ (when written using cosmic time t). In these coordinates we assume that the qubit moves along a co-moving trajectory:

其中德西特空间对应参数选择 $f(\mathbf{x}) = 1$ 和 $\gamma_{ij} = e^{2Ht}\delta_{ij}$ (使用宇宙时间 t 书写时)。在这些坐标下，我们假设量子比特沿共动轨迹运动:

$$y^\mu(\tau) = [t(\tau), \mathbf{x}(\tau)] = [\tau, \mathbf{x}_0]\,. \tag{107}$$

We take the scalar quantum field describing the quantum environment to be governed by the Klein-Gordon Hamiltonian, which using the metric (106) becomes

我们将描述量子环境的标量量子场的动力学由克莱因-戈登哈密顿量支配，代入度规 (106) 后可得

$$H_B = \frac{1}{2}\int_{\Sigma_t} \mathrm{d}^3x\sqrt{f\gamma}\left[\frac{(\partial_t\phi)^2}{f} + \gamma^{ij}\partial_i\phi\partial_j\phi + \left(m^2 - \xi R\right)\phi^2\right] \tag{108}$$

with $R = 12H^2$ the Ricci scalar computed using (96) and the integration is over a spacelike hypersurface $\sum_t$ of fixed t . We introduce here a non-minimal coupling to gravity parameterized by the dimensionless coupling ξ and assume the scalar is prepared in the Bunch-Davies vacuum $|\Omega\rangle$.

其中 $R = 12H^2$ 是由 (96) 计算得到的里奇标量，积分在固定 t 的类空超曲面 $\sum_t$ 上进行。我们在此引入一个非最小引力耦合，由无量纲耦合常数 ξ 参数化，并假设标量场初始处于邦奇-戴维斯真空 $|\Omega\rangle$ 。

With these choices the calculation of qubit response proceeds much as before. The autocorrelations of ϕ along the qubit worldline are

完成上述设定后，量子比特响应的计算过程和此前基本一致。ϕ 沿量子比特世界线的自相关函数为

$$W(\tau) = \langle \Omega | \phi[y(\tau)] \phi[y(0)] | \Omega \rangle = \langle \Omega | \phi(\tau, \mathbf{x}_0) \phi(0, \mathbf{x}_0) | \Omega \rangle \tag{109}$$

$$= \frac{H^2\left(\frac{1}{4} - v^2\right)}{16\pi\cos(\pi v)} 2F_1\left(\frac{3}{2} + v, \frac{3}{2} - v; 2; 1 + \left[\sinh\left(\frac{H\tau}{2}\right) - i\varepsilon\right]^2\right),$$

where ${}_2F_1(a, b; c; z)$ is Gauss' hypergeometric function and

其中 ${}_2F_1(a, b; c; z)$ 是高斯超几何函数，且

$$v := \sqrt{\frac{9}{4} - \frac{M^2}{H^2}} \text{ with effective mass } M^2 := m^2 - 12\xi H^2. \tag{110}$$

The late-time behaviour of this correlator is

该关联函数的迟时行为为

$$W(\tau) \simeq W_0 e^{-\kappa\tau} \text{ when } \kappa\tau \gg 1, \tag{111}$$

where

其中

$$W_0 := \frac{H^2}{4\pi^{5/2}} i e^{i\pi v} \Gamma\left(\frac{3}{2} - v\right) \Gamma(v) \text{ and } \kappa := \left(\frac{3}{2} - v\right) H \tag{112}$$

with $\Gamma(z)$ denoting Euler's gamma function.

$\Gamma(z)$ 为欧拉伽马函数。

If the qubit is prepared in its ground state then the purely perturbative rate with which it becomes excited gives the standard result for an Unruh-DeWitt detector: $\partial_\tau \rho_{\uparrow\uparrow} \simeq g^2 R(\omega)$ with $R(\omega)$ again defined by the first equality in (38), which in this case evaluates to

若量子比特初始处于基态，其被激发的纯微扰率给出了安鲁-德维特探测器的标准结果：$\partial_\tau \rho_{\uparrow\uparrow} \simeq g^2 R(\omega)$，其中 $R(\omega)$ 仍由 (38) 中的第一个等式定义，在本情形下计算可得

$$R(\omega) = \frac{H}{4\pi^3} e^{-\pi\omega/H} \left| \Gamma\left(\frac{3}{4} + \frac{v}{2} + \frac{i\omega}{2H}\right) \Gamma\left(\frac{3}{4} - \frac{v}{2} + \frac{i\omega}{2H}\right) \right|^2. \tag{113}$$

This perturbative result again breaks down at late times - applying only when $g^2 C(\omega)\tau \ll 1$ where $C(\omega)$ is defined by the first equality of (47).

该微扰结果在迟时阶段同样失效——仅当 $g^2C(\omega)\tau \ll 1$ 时成立，其中 $C(\omega)$ 由 (47) 的第一个等式定义。

Late-time evolution can again be resummed, at least for timescales long compared to the characteristic time κ^{-1} set by the fall-off of environmental correlations (111). The domain of validity of the late-time Markovian regime turns out to be more delicate because of the dependence of κ on the effective mass $M.\upsilon$ becomes imaginary if $M \gtrsim H$ and

迟时演化同样可以重求和，至少在远长于环境关联衰减 (111) 设定的特征时间 κ^{-1} 的时间尺度上成立。迟时马尔可夫区的有效范围问题更为微妙，因为当 $M \gtrsim H$ 且满足下式时，κ 对有效质量 $M.\upsilon$ 的依赖会导致 κ 变为虚数

$$\mathrm{Re}\,\kappa = \frac{3H}{2}. \tag{114}$$

The Markovian regime ends up being restricted to the parameter regime

马尔可夫区最终被限制在如下参数范围内

$$1 \gg \frac{2\pi\omega}{H} \gg \frac{g^2}{2\pi} \tag{115}$$

for reasons similar to the ones given for the uniformly accelerated qubit in (80). This results in the late-time solutions (48) and (53), describing equilibration with the Gibbons-Hawking temperature over timescales

原因和 (80) 中均匀加速量子比特的情形类似。这给出了迟时解 (48) 和 (53)，描述了量子比特在如下时间尺度上与吉本斯-霍金温度达到平衡

$$\xi_c = 2\xi_d = \frac{1}{g^2C(\omega)} \simeq \frac{4\pi^3}{g^2H}\left|\Gamma\left(\frac{3}{4}+\frac{\upsilon}{2}\right)\Gamma\left(\frac{3}{4}-\frac{\upsilon}{2}\right)\right|^{-2}. \tag{116}$$

The behaviour is very different if one instead chooses $M/H \ll 1$ (and so $\upsilon \simeq \frac{3}{2}$), since in this limit

如果转而选择 $M/H \ll 1$ (因此也得到 $\upsilon \simeq \frac{3}{2}$)，行为会截然不同，因为在这个极限下

$$W_0 \simeq \frac{3H^4}{8\pi^2M^2} \quad \text{and} \quad \kappa \simeq \frac{M^2}{3H}. \tag{117}$$

Both the amplitude and width of the correlation function $W(\tau)$ become parametrically large in the limit of small effective mass. Although Markovianity ultimately applies, it does so (for $M/H \ll 1$) only in the more restrictive regime

在小有效质量极限下，关联函数 $W(\tau)$ 的振幅和宽度都会在参数层面变得很大。尽管马尔可夫性最终仍然成立，但 (对于 $M/H \ll 1$ 来说) 它仅在限制更强的区域成立

$$1 \gg \frac{\omega}{H} \gg \frac{M^2}{H^2} \quad \text{and} \quad \frac{M^6}{H^6} \gg g^2 \tag{118}$$

with relaxation timescales now taking the form

此时弛豫时间尺度取如下形式

$$\xi_c = 2\xi_d = \frac{1}{g^2 C(\omega)} \simeq \frac{4\pi^2 M^4}{9g^2 H^5}. \tag{119}$$

Notice that $\xi \gg 1/\kappa$ in this regime because the domain of validity (118) ensures the $1/g^2$ enhancement overwhelms the M/H suppression. The Markovian approximation is so restrictive in this instance because it requires the width of $W(\tau)$ to be the shortest timescale in the problem, and this means that the enormous timescale $1/\kappa \simeq 3H/M^2$ must be smaller than any of the other scales associated with qubit evolution (like ξ and $1/\omega$).

请注意，在该区域存在 $\xi \gg 1/\kappa$ ，因为有效域 (118) 保证了 $1/g^2$ 的增强效果盖过了 M/H 的抑制效果。在这种情况下马尔可夫近似的限制很强，因为它要求 $W(\tau)$ 的宽度是该问题中最短的时间尺度，这意味着极长的时间尺度 $1/\kappa \simeq 3H/M^2$ 必须小于和量子比特演化相关的所有其他尺度 (比如 ξ 和 $1/\omega$)。

We note in passing that it is also possible to solve explicitly for qubit evolution at late times even when this evolution is non-Markovian by returning to the Nakajima-Zwanzig equation (35) and (36). This more cumbersome calculation enlarges the domain of validity for which resummed late-time evolution can be obtained - for instance applying in a regime where $\omega/H \ll 1$ can be either larger or smaller than $M/H \ll 1$, unlike in (118) above (see [36] for details).

我们顺便指出，即便当演化是非马尔可夫的，也可以通过回到中岛-茨维基方程 (35) 和 (36)，显式求解晚期的量子比特演化。这个更繁琐的计算扩大了可得到重求和晚期演化的有效域——例如，它适用于 $\omega/H \ll 1$ 可大于也可小于 $M/H \ll 1$ 的区域，不同于上述的 (118)(详情见 [36])。

Coarse-Grained Fields

粗粒化场

We close this section with several examples of Open EFT calculations for which both system and environment are described by fields. Although this gives up the simplicity of the qubit examples, many of the lessons learned there continue to go through. In the examples considered we take the observed system to consist of super-Hubble modes, for which $k/a \ll H$, and seek the influence on these due to shorter-wavelength modes.

我们以几个开放有效场论 (Open EFT) 计算例子结束本节，这些例子中系统与环境均由场描述。尽管这会失去量子比特例子的简洁性，从量子比特例子中得到的许多结论仍然成立。在我们考察的例子中，将观测系统取为超哈勃模式，满足 $k/a \ll H$ ，并研究短波长模式对这些模式的影响。

We describe ongoing work aimed at two kinds of applications: one outlining the late-time evolution of the probability distribution for the amplitude of super-Hubble scalar field modes; the other computing the decoherence rate of field fluctuations during inflation, initially of a spectator scalar field but eventually for metric fluctuations more generally.

我们介绍目前针对两类应用开展的工作: 一类是描绘超哈勃标量场模式振幅概率分布的晚期演化; 另一类是计算暴胀期间场涨落的退相干速率, 最初针对旁观者标量场, 最终推广到一般的度规涨落。

Consider first a spectator scalar field Φ (i.e. one whose energy density is negligible relative to the energy density responsible for the curvature of the de Sitter geometry). Expanding about a background configuration: $\Phi(t, \mathbf{x}) = \varphi(t) + \phi(t, \mathbf{x})$ on a near-de Sitter metric, the system and environment are defined in terms of short- and long-wavelength modes - cf. Eq. (100) - so $\phi(x) = \phi_{\text{sys}}(x) + \phi_{\text{env}}(x)$ with

首先考虑旁观者标量场 Φ (即它的能量密度远小于造成德西特几何曲率的能量密度)。对近德西特度规下的背景构型 $\Phi(t, \mathbf{x}) = \varphi(t) + \phi(t, \mathbf{x})$ 做展开，系统和环境按短波长模式与长波长模式定义——参见式 (100)——因此有 $\phi(x) = \phi_{\text{sys}}(x) + \phi_{\text{env}}(x)$，其中

$$\phi_{\text{sys}}(x) := \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} \left[v_{\mathbf{k}}(x) \mathfrak{a}_{\mathbf{k}} + v^*_{\mathbf{k}}(x) \mathfrak{a}^*_{\mathbf{k}} \right] f(k, k_*)$$

$$\text{and } \phi_{\text{env}}(x) := \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} \left[v_{\mathbf{k}}(x) \mathfrak{a}_{\mathbf{k}} + v^*_{\mathbf{k}}(x) \mathfrak{a}^*_{\mathbf{k}} \right] [1 - f(k, k_*)] \,, \tag{120}$$

and $0 \le f(k, k_*) \le 1$ a window function that distinguishes ’short’ from ’long ‘wavelengths relative to a reference k_* .

且 $0 \le f(k, k_*) \le 1$ 为窗函数，以参考尺度 k_* 为界区分“短”波长和“长”波长。

For instance if $f(k, k_*) = \Theta(k_* - k)$ is a Heaviside step function then the system consists of those modes whose co-moving momentum satisfies $k < k_*$. (More generally, smoother choices for f that transition from 0 to 1 over a region $k_* - \delta < k < k_* + \delta$ could also be entertained.) All modes with $k < k_*$ are super-Hubble after some time t_0 where $p_*(t_0) := k_*/a(t_0) < H$. Alternatively, it can also be convenient for some purposes to allow $f(k, k_*)$ also to depend on t , such as if the system/environment split is defined in terms of physical wavelengths. For instance, if the system is defined by $p(t) < \Lambda$ for all t where Λ is a fixed physical scale then $f(k, \Lambda, t) = \Theta[\Lambda a(t) - k]$. In such circumstances the derivation given in section ”Open Quantum Systems” for the time evolution of the system density matrix $\rho_A(t)$ must be revisited to allow for a time-dependent division between system and environment.

例如，若 $f(k, k_*) = \Theta(k_* - k)$ 是海维赛德阶跃函数，则系统由共动动量满足 $k < k_*$ 的模式构成。(更一般地，也可以采用更平滑的 f 选择，在区域 $k_* - \delta < k < k_* + \delta$ 上从 0 过渡到 1。) 所有满足 $k < k_*$ 的模式都会在时刻 t_0 之后成为超哈勃模式，其中 $p_*(t_0) := k_*/a(t_0) < H$ 。此外，在某些情况下，允许 $f(k, k_*)$ 也依赖于 t 会更方便，比如按物理波长定义系统/环境划分的情形: 如果系统由条件 $p(t) < \Lambda$ 定义，对所有 t ，其中 Λ 是固定物理尺度，那么可得 $f(k, \Lambda, t) = \Theta[\Lambda a(t) - k]$ 。在这种情况下，需要重新推导“开放量子系统”一节给出的系统密度矩阵 $\rho_A(t)$ 时间演化公式，以适配系统与环境之间的划分随时间变化的情况。

Stochastic Inflation

随机暴胀

The evidence for the existence of secular effects is clearer for de Sitter geometries because explicit calculations of subleading perturbative effects have been performed. This section summarizes an example, together with the preliminary evidence that the secular growth visible in it can be resummed [37] using the formalism of Stochastic Inflation [38]. The story of how stochastic inflation itself is now becoming understood as the leading part of a more systematic approximation is the topic of Chap. 5, "EFT for de Sitter Space".

对于德西特几何，长期效应存在的证据更为清晰，因为次领头阶微扰效应的显式计算已经完成。本节总结了一个例子，以及初步证据表明其中可见的长期增长可以用随机暴胀形式体系 [38] 进行重求和 [37]。随机暴胀本身如今如何被理解为更系统近似的领头部分，这一内容是第 5 章“德西特空间的有效场论”的主题。

To this end consider again massless $\lambda\phi^4$ theory, with action as given by (54), specialized to the FLRW metric (95) with de Sitter scale factor (98). For simplicity take the stress energy associated with the scalar field to be much smaller than the value of the cosmological constant responsible for the de Sitter curvature - i.e. a 'spectator' field. The consistency of this assumption can be verified ex post facto by checking that the scalar stress energy in the state of interest is order H^4 . See however [39, 40] for the treatment of non-spectator scalars whose stress energy drives inflation, and [41] for the extension of spectator scalars to include scalar masses and slow-roll corrections.

为此我们再次考虑无质量 $\lambda\phi^4$ 理论，其作用量如式 (54) 所示，专门应用于带有德西特尺度因子 (98) 的 FLRW 度规 (95)。为简化起见，假设标量场对应的应力能量远小于产生德西特曲率的宇宙学常数，即该标量场是一个“旁观”场。该假设的自洽性可以事后验证: 只需检查目标态中标量场的应力能量为 H^4 量级。关于非旁观标量 (其应力能量驱动暴胀) 的处理见 [39, 40]，关于将旁观标量推广到包含标量质量和慢滚修正的内容见 [41]。

$O(\lambda)$ corrections have been computed explicitly for $\langle\phi^2(x)\rangle$ evaluated in the adiabatic (Bunch-Davies) vacuum whose mode functions are given in (102). For a massive scalar field the symmetries of de Sitter space (and the Bunch-Davies vacuum) ensure $\langle\phi^2(x)\rangle$ is independent of x and given by (103) (so is singular [12] as $m \to 0$). As discussed around Eq. (104), this singularity shows up in the massless limit as an IR divergence in $\langle\phi^2(x)\rangle$ and depending on how it is regulated [13] this can introduce a time-dependence to $\langle\phi^2\rangle$ in the massless limit. For instance (105) gives $\langle\phi^2\rangle \propto t = H^{-1}\ln a$ and including $O(\lambda)$ corrections turns out to give [37]

对于在绝热 (邦奇-戴维斯) 真空中计算的 $\langle\phi^2(x)\rangle$，$O(\lambda)$ 修正已经被显式算出，该真空的模函数由式 (102) 给出。对于有质量标量场，德西特空间 (以及邦奇-戴维斯真空) 的对称性保证 $\langle\phi^2(x)\rangle$ 与 x 无关，其表达式为 (103)(因此当 $m \to 0$ 时，[12] 处发散)。正如式 (104) 附近讨论的，该奇点在无质量极限下表现为 $\langle\phi^2(x)\rangle$ 中的红外发散，根据对其正规化的方式 [13]，这会在无质量极限下给 $\langle\phi^2\rangle$ 引入时间依赖性。例如式 (105) 给出 $\langle\phi^2\rangle \propto t = H^{-1}\ln a$，纳入 $O(\lambda)$ 修正后得到结果 [37]:

$$\langle\phi^2(x)\rangle = \frac{H^2}{4\pi^2}\ln a\left[1 - \frac{\lambda}{36\pi^2}\ln^2 a + \cdots\right], \tag{121}$$

up to an additive constant whose value is regularization dependent but irrelevant for the time-dependence of the right-hand side. The factor of $\ln^2 a = (Ht)^2$ multiplying λ is the secular growth that undermines trust in perturbative methods at late time.

结果保留一个加法常数，该常数的值依赖正规化方案，但对右侧的时间依赖性没有影响。乘在 λ 前的 $\ln^2 a = (Ht)^2$ 因子就是长期增长，它导致微扰方法在晚期不可信。

Besides showing the existence of secular growth, this same calculation also provides evidence that this growth can be controllably resummed. The evidence comes from comparing the (IR finite) derivative $\partial_t \left\langle \phi^2 (x) \right\rangle$ computed from (121) with the predictions of Stochastic Inflation [38]. Stochastic Inflation starts with the similarity between the leading prediction $\left\langle \phi^2 (t) \right\rangle \propto t$ and the variance of the distance travelled in a random walk. The proposal is to compute the time evolution of field correlations on super-Hubble scales by building on this random-walk analogy; by regarding the approximately position-independent value φ taken by the super-Hubble part of the field to be a random variable in the region of size H^{-1} surrounding x (a ’ Hubble patch’). If $P(\varphi, t)$ denotes the probability of it taking the value φ at time t then correlators can be computed using formulae like

除了证明长期增长的存在, 这一计算还提供了该增长可以被可控重求和的证据。证据来自将式 (121) 算出的 (红外有限的) 导数 $\partial_t \left\langle \phi^2 (x) \right\rangle$ 与随机暴胀 [38] 的预言进行对比。随机暴胀始于领头预言 $\left\langle \phi^2 (t) \right\rangle \propto t$ 与随机游走行走距离的方差之间的相似性。该方案基于这种随机游走类比来计算哈勃视界外尺度上场关联的时间演化: 将场的超哈勃部分近似为位置无关的值 φ， 将该值看作围绕 x 的大小为 H^{-1} 区域 (即“哈勃 patch”) 中的随机变量。若 $P(\varphi, t)$ 表示时刻 t 场取值为 φ 的概率，那么关联函数可以用如下公式计算:

$$\left\langle \phi^{2n} (t) \right\rangle = \int \mathrm{d}\varphi \varphi^{2n} P(\varphi, t) . \tag{122}$$

The random-walk part of the picture enters when computing the time-dependence of $\left\langle \phi^{2n} (t) \right\rangle$, with the time evolution of $P(\varphi, t)$ taken to be governed by a Fokker-Planck equation

当计算 $\left\langle \phi^{2n} (t) \right\rangle$ 的时间依赖性时，图像中的随机游走部分就会发挥作用，此时 $P(\varphi, t)$ 的时间演化由福克-普朗克方程控制:

$$\partial_t P = \frac{H^3}{8\pi^2} \frac{\partial^2 P}{\partial \varphi^2} + \frac{1}{3H} \frac{\partial}{\partial \varphi} \left(\frac{\partial V}{\partial \varphi} P \right), \tag{123}$$

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[12] Notice the same is not true for the energy density since $\left\langle m^2 \phi^2 \right\rangle \sim H^4$.

[12] 请注意，能量密度的情况并非如此，因为 $\left\langle m^2 \phi^2 \right\rangle \sim H^4$ 。

[13] Introducing a small nonzero mass is an example of a time-independent IR regularization.

[13] 引入一个微小的非零质量就是与时间无关的红外正则化的一个例子。

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as one would expect for a random walk in the presence of a potential $V(\varphi)$. The second term of the right-hand side of this equation is designed to properly evolve the mean

正如我们对存在势场 $V(\varphi)$ 的随机游走所预期的那样。该方程右侧的第二项用于正确演化平均值

$$\partial_t\langle\phi(t)\rangle = \int d\varphi\varphi\partial_t P(\varphi, t) = -\frac{1}{3H}\langle V'(\phi)\rangle \tag{124}$$

corresponding to the evolution $3H\dot{\phi} + V'(\phi) = 0$. This is the evolution equation for ϕ that would be obtained from (99) if a potential $V(\phi)$ were added and if restricted to the 'slow-roll' regime where $\ddot{\phi}$ can be neglected.

对应于演化 $3H\dot{\phi} + V'(\phi) = 0$。这就是添加势场 $V(\phi)$ 并限制在可忽略 $\ddot{\phi}$ 的“慢滚”区域时，可从式 (99) 得到的 ϕ 演化方程。

The first term on the right-hand side of (123) is similarly chosen to reproduce the leading $H^3/(4\pi^2)$ contribution - cf. Eq. (105) - to the rate of change of the variance. For instance, specializing (123) to $V = \frac{1}{4!}\lambda\phi^4$ leads to the prediction

(123) 式右侧的第一项同理是为了重现方差变化率的主导阶 $H^3/(4\pi^2)$ 贡献——参见式 (105)。例如，将 (123) 特殊化到 $V = \frac{1}{4!}\lambda\phi^4$ 可得到预测

$$\partial_t\left\langle\phi^2(t)\right\rangle = \int d\varphi\varphi^2\partial_t P(\varphi, t) = \frac{H^3}{4\pi^2} - \frac{\lambda}{9H}\left\langle\phi^4\right\rangle. \tag{125}$$

Reference [37] sets up the hierarchy of evolution equations for $\partial_t\left\langle\phi^{2n}(t)\right\rangle$ implied by (123) by continuing as above for the specific case $V = \frac{1}{4!}\lambda\phi^4$ and then solves the resulting recursion relations. This leads to the more explicit predictions

参考文献 [37] 针对特定情形 $V = \frac{1}{4!}\lambda\phi^4$ 沿用上述推导，建立了由 (123) 导出的 $\partial_t\left\langle\phi^{2n}(t)\right\rangle$ 演化方程组，随后求解了得到的递推关系，给出了更明确的预言

$$\left\langle\phi^{2n}(t)\right\rangle = (2n-1)!!\left(\frac{H^2}{4\pi^2}\ln a\right)^n\left[1 - \frac{n(n+1)}{2}\left(\frac{\lambda}{36\pi^2}\right)\ln^2 a + \right. \tag{126}$$
$$\left. + \frac{n}{280}\left(35n^3 + 170n^2 + 225n + 74\right)\left(\frac{\lambda}{36\pi^2}\ln^2 a\right)^2 + \cdots\right].$$

This expression agrees - including $O(\lambda)$ corrections - with results like (121) for the evolution of these quantities predicted by explicit field theory calculations. This suggests that the stochastic formulation captures the long-wavelength part of the perturbative result, potentially giving insight into how small secular effects evolve. Furthermore, it does so in a way that seems to give access to the late-time future towards which the secular evolution ultimately leads. In the stochastic formulation the evolution describes relaxation towards a static state, whose form can be found by solving (123) for $P_\infty(\varphi)$ under the assumption that $\partial_t P_\infty = 0$. This leads to

该表达式与显式场论计算预测的这些量演化结果 (比如式 (121)) 一致，包括 $O(\lambda)$ 修正。这说明随机表述抓住了微扰结果的长波长部分，有助于理解长期 secular 效应如何演化。此外，该表述还能让我们触及长期演化最终走向的未来晚期阶段。在随机表述中，演化描述了系统向静态弛豫的过程，假设 $\partial_t P_\infty = 0$ 成立时求解 (123) 式得到 $P_\infty(\varphi)$，即可得到该静态的形式，结果为

$$\frac{H^3}{8\pi^2}\frac{\partial^2 P_\infty}{\partial\varphi^2}+\frac{1}{3H}\frac{\partial}{\partial\varphi}\left(\frac{\partial V}{\partial\varphi}P_\infty\right)=0, \tag{127}$$

with late-time solution

其晚期解为

$$P_\infty(\varphi)=C\exp\left[-\frac{8\pi^2 V(\varphi)}{3H^4}\right]. \tag{128}$$

In the case of a free massive field $V=\frac{1}{2}m^2\varphi^2$ Eq. (128) describes a Gaussian distribution around mean $\langle\phi\rangle=0$ with the correct variance $\langle\phi^2\rangle=3H^4/\left(8\pi^2 m^2\right)$. But for an interacting potential $V=\frac{1}{4!}\lambda\varphi^4$ Eq. (128) instead predicts evolution towards a very non-Gaussian distribution.

对于自由大质量场 $V=\frac{1}{2}m^2\varphi^2$，式 (128) 描述了均值为 $\langle\phi\rangle=0$、方差为 $\langle\phi^2\rangle=3H^4/\left(8\pi^2 m^2\right)$ 的高斯分布，符合预期。但对于相互作用势 $V=\frac{1}{4!}\lambda\varphi^4$，式 (128) 反而预测演化会朝向高度非高斯分布。

Considerable effort has been devoted to proving that late-time evolution of quantum fields on de Sitter space is well-described by Stochastic Inflation [38, 42, 43], and in particular how it might emerge more systematically as the leading approximation for long-wavelength modes within an open-system approach along the lines used here [44]. Recent efforts have developed diagrammatic arguments to identify more systematically both how the stochastic limit arises and what its leading corrections are [45]. We defer to Chap. 5, ”EFT for de Sitter Space” for a more expert description of these developments.

学界已做了大量研究证明，德西特空间上量子场的晚期演化可以被随机暴胀很好地描述 [38, 42, 43]，尤其是在本文采用的开系方法框架下，它如何能更系统地成为长波长模式的主导近似 [44]。近年的研究通过图论论证更系统地阐明了随机极限的起源及其主导修正 [45]。我们将在第 5 章“德西特空间的有效场论”中更专业地介绍这些进展。

Primordial Decoherence

原初退相干

We close this section with a variation on the above themes that describes a more practical field-theoretic Open EFT calculation. We apply this formalism to compute the decoherence rate of primordial fluctuations during inflation. More specifically, we describe the speed with which gravitational self-interactions can allow unseen short-wavelength metric fluctuations to decohere their observed longer wavelength cousins that are believed to seed primordial fluctuations within inflationary cosmologies (see also [46-51]).

我们围绕上述主题的一种变体结束本节，该变体描述了更实用的场论开有效场论计算。我们应用这套形式体系计算暴胀期间原初涨落的退相干率。更具体来说，我们阐述引力自相互作用使不可见短波长度规涨落让可观测长波长涨落退相干的速率——而长波长涨落被认为是暴胀宇宙学中原初涨落的种子 (另见文献 [46-51])。

To this end consider the following single-field inflationary model with inflaton Φ coupled to gravity with action

为此我们考察下述单场暴胀模型: 暴胀子 Φ 与引力耦合，其作用量为

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V(\Phi)\right], \tag{129}$$

where $V(\Phi)$ is designed so the classical homogeneous solutions $\varphi(t)$ describe slow-roll inflation, and so in particular ensure $\varepsilon_1(\varphi) := -\dot{H}/H^2 \simeq \frac{1}{2}\left(M_p\partial_\phi V/V\right)^2 \ll 1$. The scalar is not assumed to be a spectator and as a result fluctuations about the background mix scalar and metric modes. Writing $\Phi(\mathbf{x},t) = \varphi(t) + \phi(\mathbf{x},t)$ and expanding the metric

其中 $V(\Phi)$ 的构造使得经典均匀解 $\varphi(t)$ 描述慢滚暴胀，且特别满足 $\varepsilon_1(\varphi) := -\dot{H}/H^2 \simeq \frac{1}{2}\left(M_p\partial_\phi V/V\right)^2 \ll 1$。该标量不被假定为旁观者，因此背景涨落会混合标量模式与度规模式。记 $\Phi(\mathbf{x},t) = \varphi(t) + \phi(\mathbf{x},t)$ 并将度规展开

$$ds^2 = -N^2\,dt^2 + h_{ij}\left(dx^i + N^i\,dt\right)\left(dx^j + N^j\,dt\right), \tag{130}$$

standard arguments allows us to write the metric fluctuation to second order as

标准推导可以让我们将度规涨落写至二阶，形式为

$$h_{ij} = a^2 e^{2\zeta}\hat{h}_{ij} \;\text{ with }\; \hat{h}_{ij} = \delta_{ij} + \gamma_{ij} + \frac{1}{2}\delta^{kl}\gamma_{ik}\gamma_{lj} + \cdots, \tag{131}$$

where $a(t)$ is the near-de Sitter scale factor for the background FLRW metric, $\det\hat{h}_{ij} = 1$ and $\delta^{ij}\partial_i\gamma_{jk} = \delta^{ij}\gamma_{ij} = 0$.

其中 $a(t)$ 是背景 FLRW 度规的近德西特尺度因子，$\det\hat{h}_{ij} = 1$ 且 $\delta^{ij}\partial_i\gamma_{jk} = \delta^{ij}\gamma_{ij} = 0$ 。

γ_{ij} describes the metric's tensor perturbations (gravitational waves) while one combination of ϕ and ζ describes the metric's scalar perturbations and the other combination represents a gauge freedom corresponding to different ways to foliate the spacetime into time slices. Two convenient gauge conditions are the choices $\phi = 0$ (co-moving gauge) or $\zeta = 0$ (spatially flat gauge).

γ_{ij} 描述度规的张量微扰 (引力波)，ϕ 与 ζ 的一种组合描述度规的标量微扰，另一种组合对应规范自由度，即不同的时空时间切片分层方式。两种方便的规范条件是取 $\phi = 0$ (共动规范) 或 $\zeta = 0$ (空间平直规范)。

In co-moving gauge the leading (quadratic) part of the action governing fluctuations has the form [52, 53]

在共动规范下，描述涨落的作用量的领头 (二次) 项形式为 [52, 53]

$${}^{(2)}S = \int dt\, d^3x\left\{\frac{\dot{\varphi}^2}{2H^2}\left[a^3\dot{\zeta}^2 - a(\partial\zeta)^2\right] + \frac{M_p^2}{8}\left[a^3\dot{\gamma}^{ij}\dot{\gamma}_{ij} - a\left(\partial^k\gamma^{ij}\right)\left(\partial_k\gamma_{ij}\right)\right]\right\}, \tag{132}$$

where spatial indices are raised and lowered using δ_{ij} , so $(\partial\zeta)^2 = \delta^{ij}\partial_i\zeta\partial_j\zeta$. This has the canonical form $\frac{1}{2}\int d\eta\left[(v')^2 + \left(v'_{ij}\right)^2 + \cdots\right]$ once rewritten in terms of the Mukhanov-Sasaki variables

其中空间指标用 δ_{ij} 升降，因此 $(\partial\zeta)^2 = \delta^{ij}\partial_i\zeta\partial_j\zeta$ 。改写为穆哈诺夫-佐佐木变量后，它就得到标准形式 $\frac{1}{2}\int d\eta\left[(v')^2 + \left(v'_{ij}\right)^2 + \cdots\right]$

$$v(\eta,\mathbf{x}) = aM_p\sqrt{2\varepsilon_1}\zeta(\eta,\mathbf{x}) \quad \text{and} \quad v_{ij}(\eta,\mathbf{x}) = \frac{1}{2}aM_p\gamma_{ij}(\eta,\mathbf{x}), \tag{133}$$

where the slow-roll parameter enters because of the background classical slow-roll relation $\dot{\varphi}^2 = 2H^2M_p^2\varepsilon_1$.

其中慢滚参数的引入来自背景经典慢滚关系 $\dot{\varphi}^2 = 2H^2M_p^2\varepsilon_1$ 。

Interaction terms are obtained by expanding the action (129) to cubic and higher order in the fluctuations, and every extra power of v or v_{ij} costs a power of $1/M_p$. For instance, at cubic order one finds the interactions [54]

相互作用项可通过将作用量 (129) 按涨落展开至三阶及更高阶得到，v 或 v_{ij} 每多出一次幂，就带来一个 $1/M_p$ 幂次的压低。例如，三阶可得到相互作用项 [54]

$$\begin{aligned} {}^{(3)}S &= \int dt\, d^3x M_p^2\left[\varepsilon_1^2 a\zeta(\partial\zeta)^2 + \varepsilon_1 a\gamma^{ij}\partial_i\zeta\partial_j\zeta + \frac{\varepsilon_1}{8}a\zeta\partial^l\gamma^{ij}\partial_l\gamma_{ij} + \cdots\right] \\ &= \int \frac{d\eta d^3x}{aM_p}\left\{\frac{\sqrt{\varepsilon_1}}{2\sqrt{2}}v\left[(\partial v)^2 + \partial^l v^{ij}\partial_l v_{ij}\right] + v^{ij}\partial_i v\partial_j v + \cdots\right\}, \end{aligned} \tag{134}$$

where the ellipses involve numerous other interactions, all of which either do not involve the scalar mode v , or are suppressed by more slow-roll parameters or involve more time derivatives than the ones explicitly shown. Further terms involving quartic and higher powers of v and v_{ij} are also present, suppressed by at least two powers of $1/M_p$, and so on.

其中省略号包含诸多其他相互作用项，这些相互作用要么不包含标量模式 v ，要么被更多慢滚参数压低，要么比我们明确写出的项包含更多时间导数。还存在包含 v 和 v_{ij} 四次及更高次幂的额外项，这些项至少被 $1/M_p$ 的两个幂次压低，以此类推。

From here the goal is to split the fields into system and environment components, where the system consists of those modes whose wavelengths are visible to observers in the late universe. We therefore follow [55] and split the fields v and v_{ij} as in (120), with the environment/system split occurring at a scale k_* chosen as the shortest modes currently accessible in late-time cosmology. We compute how these modes evolve while outside the Hubble scale during the tail end of inflation, focussing on how the system modes are decohered by the shorter-wavelength environment. This involves tracking the evolution of the off-diagonal components $\langle\varphi_1|\rho|\varphi_2\rangle$ of the reduced density matrix rather than the diagonal ones $P(\varphi) = \langle\varphi|\rho|\varphi\rangle$ whose evolution is relevant to the validity of Stochastic Inflation discussed above.

从此处开始，我们的目标是将场划分为系统分量和环境分量，其中系统由晚期宇宙中的观测者可观测到波长的模式构成。因此我们遵循文献 [55]，按照 (120) 式拆分场 v 和 v_{ij} ，环境/系统的划分发生在标度 k_* 处，该标度被选为当前晚期宇宙学可及的最短模式。我们计算这些模式在暴胀末期处于哈勃尺度之外时的演化，重点关注系统模式如何被短波长环境退相干。这一过程需要跟踪约化密度矩阵的非对角分量 $\langle\varphi_1|\rho|\varphi_2\rangle$ ，而非对角分量 $P(\varphi) = \langle\varphi|\rho|\varphi\rangle$ ——对角分量的演化与前文讨论的随机暴胀的有效性相关。

Inspection of the general evolution equation (22) shows that decoherence first arises at second order in the interaction that couples system to environment, and at lowest order in $1/M_p$ the relevant interaction are cubic in the fields, with the fields split into system and environment parts: $v = v_{\text{sys}} + v_{\text{env}}$. Because only primordial scalar fluctuations have been observed we focus on how these decohere and so can ignore those cubic interactions not involving v . Interactions involving more than the minimal slow-roll suppression can also be dropped as subdominant. Finally, the freezing of super-Hubble modes implies that time derivatives can also be dropped relative to spatial derivatives in all interactions provided we focus on super-Hubble environmental modes. What remains are then only the interactions given by (134). Not all of these interactions are even required because derivatives acting on system fields are always suppressed relative to their shorter-wavelength cousins in the environment. Momentum conservation also precludes having only one environment field since one large momentum cannot sum with two small ones to give zero.

考察一般演化方程 (22) 可知，退相干首先出现在系统与环境耦合相互作用的二阶项，在 $1/M_p$ 的最低阶，相关相互作用是场的三次型，场已划分为系统部分和环境部分: $v = v_{\text{sys}} + v_{\text{env}}$ 。由于仅观测到原初标量涨落，我们聚焦于这些涨落的退相干过程，因此可以忽略不包含 v 的三次相互作用。涉及超过最小慢滚压低的相互作用也可作为次主导项丢弃。最后，超哈勃模式的冻结意味着，如果我们关注的是超哈勃环境模式，那么在所有相互作用中，相对于空间导数，时间导数也可以丢弃。最终仅剩 (134) 式给出的相互作用。并非所有这些相互作用都是必需的，因为作用在系统场上的导数，相对于环境中的短波长同类项总是被压低。动量守恒也排除了仅存在一个环境场的情况，因为一个大动量无法与两个小动量加和为零。

In the end the only interactions that matter to leading order are the first two appearing in (134), where the differentiated fields are environmental modes and the undifferentiated fields belong to the system. Of these, the first interaction mediates decoherence of long-wavelength scalar fluctuations by the short-wavelength scalar environment and the second interaction describes decoherence of long-wavelength scalar fluctuations by the short-wavelength tensor environment. The interaction Hamiltonian to be used in (22) therefore becomes

最终，对主导阶有贡献的相互作用只有 (134) 式中出现的前两项: 其中带导数的场是环境模式，不带导数的场属于系统。在这两项中，第一项介导长波标量涨落在短波标量环境下的退相干，第二项描述长波标量涨落在短波张量环境下的退相干。因此，可用于 (22) 式的相互作用哈密顿量为

$$\mathcal{H}_{\text{int}}(\eta) = G(\eta) \int \mathrm{d}^3x v_{\text{sys}}(\eta, \mathbf{x}) \otimes \left[B^S(\eta, \mathbf{x}) + B^T(\eta, \mathbf{x})\right], \tag{135}$$

where the effective coupling and the scalar and tensor environmental interaction operators are

其中有效耦合以及标量和张量环境相互作用算符为

$$G(\eta) := -\frac{\sqrt{\varepsilon_1}}{2\sqrt{2}M_p a}, \; B^S := \partial^i v_{\rm env} \partial_i v_{\rm env} \;\; \text{and} \;\; B^T := \partial^l v^{ij} \partial_l v_{ij}. \tag{136}$$

Because this interaction is linear in $v_{\rm sys}$ its use at second order in (22) gives an evolution equation for the system state that is at most quadratic in $v_{\rm sys}$. Together with momentum conservation this implies that the field state for each mode $\mathbf{k}$ evolves independent of the others. Starting in the remote past with each mode uncorrelated (as is true in particular for the Bunch-Davies state) then ensures they remain uncorrelated and allows the system's reduced density matrix to be written

由于该相互作用对 $v_{\rm sys}$ 是线性的，将其代入 (22) 式的二阶项后，得到的系统态演化方程对 $v_{\rm sys}$ 最多是二次的。结合动量守恒可知，每个模式 $\mathbf{k}$ 的场态演化独立于其他模式。从遥远过去每个模式互不关联开始 (邦奇-戴维斯态尤其满足这一点)，这保证了它们始终保持不关联，因此系统的约化密度矩阵可以写为

$$\rho_{\rm sys}(\eta) = \bigotimes_{k<k_*} \rho_{\mathbf{k}}(\eta), \tag{137}$$

with each $\rho_{\mathbf{k}}(\eta)$ evolving independently.

其中每个 $\rho_{\mathbf{k}}(\eta)$ 独立演化。

Using (135) and (137) in (22) reveals that the environmental correlation relevant to primordial scalar fluctuations is $C_{\rm env}(\eta, \eta'; \mathbf{y}) = C^S(\eta, \eta'; \mathbf{y}) + C^T(\eta, \eta'; \mathbf{y})$ where

将 (135) 和 (137) 代入 (22) 可知，与原初标量涨落相关的环境关联为 $C_{\rm env}(\eta, \eta'; \mathbf{y}) = C^S(\eta, \eta'; \mathbf{y}) + C^T(\eta, \eta'; \mathbf{y})$，其中

$$C^S(\eta, \eta'; \mathbf{x} - \mathbf{x}') := \left\langle \left[B^S(\eta, \mathbf{x}) - \mathfrak{B}^S(\eta)\right]\left[B^S(\eta', \mathbf{x}') - \mathfrak{B}^S(\eta')\right]\right\rangle, \tag{138}$$

with $\mathfrak{B}^S(\eta) := \langle B^S(\eta, \mathbf{x})\rangle$. The result for C^T is identical with the replacement $B^S \to B^T$. In these expressions expectation values for all environmental modes are taken in the Bunch-Davies state. The combination that controls the evolution of the state $\rho_{\mathbf{k}}(\eta)$ is then $C_{\mathbf{k}}(\eta, \eta')$ where

且满足 $\mathfrak{B}^S(\eta) := \langle B^S(\eta, \mathbf{x})\rangle$。对 C^T 做替换 $B^S \to B^T$ 后得到的结果完全相同。在这些表达式中，所有环境模式的期望值都是在邦奇-戴维斯态下取的。控制态 $\rho_{\mathbf{k}}(\eta)$ 演化的组合为 $C_{\mathbf{k}}(\eta, \eta')$，其中

$$C_{\rm env}(\eta, \eta'; \mathbf{y}) = \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} C_{\mathbf{k}}(\eta, \eta') e^{i\mathbf{k}\cdot\mathbf{y}}. \tag{139}$$

These correlation functions are evaluated explicitly in [55] where it is also shown that they are peaked in a way that allows approximate Markovian evolution in the super-Hubble regime $|k\eta| \ll 1$.

这些关联函数已在文献 [55] 中明确计算，文中还证明它们的峰值分布特性使得超哈勃区域可以存在近似马尔可夫演化 $|k\eta| \ll 1$。

The Markovian evolution equation to which one is led in this way is

由此推导出的马尔可夫演化方程为

$$\frac{\mathcal{V}}{(2\pi)^3}\frac{\partial \rho_{\mathbf{k}}}{\partial \eta} \simeq -\operatorname{Re}\left[\mathfrak{F}_{\mathbf{k}}(\eta,\eta_{\text{in}})\right]\left[v_{\mathbf{k}}(\eta),\left[v_{\mathbf{k}}(\eta),\rho_{\mathbf{k}}(\eta)\right]\right] \tag{140}$$

$$-i\operatorname{Im}\left[\mathfrak{F}_{\mathbf{k}}(\eta,\eta_{\text{in}})\right]\left[\left[v_{\mathbf{k}}(\eta)\right]^2,\rho_{\mathbf{k}}(\eta)\right],$$

where $\mathcal{V}$ denotes the volume of space and enters due to the way we normalize momentum modes. The coefficient function is defined by

其中 $\mathcal{V}$ 表示空间体积，它是由我们对动量模式的归一化方式引入的。系数函数定义为

$$\mathfrak{F}_{\mathbf{k}}(\eta,\eta_{\text{in}}) := (2\pi)^{3/2}\int_{\eta_{\text{in}}}^{\eta} \mathrm{d}\eta'\, G(\eta)\, G(\eta')\, C_{\mathbf{k}}(\eta,\eta'). \tag{141}$$

Explicit expressions for $\mathfrak{F}_{\mathbf{k}}$ are given in [55], which shows in particular that $\operatorname{Re}\mathfrak{F}_{\mathbf{k}}$ is UV finite. UV divergences do appear in $\operatorname{Im}\mathfrak{F}_{\mathbf{k}}$ and do so in a way that can be renormalized into parameters in the effective lagrangian in the usual EFT way (as reviewed, for example, in [34]). Because the second line of (140) describes Liouville evolution it cannot contribute to decoherence, which therefore depends only on $\operatorname{Re}\mathfrak{F}_{\mathbf{k}}$ and as a consequence is UV finite.

$\mathfrak{F}_{\mathbf{k}}$ 的显式表达式已在文献 [55] 中给出，其中特别证明了 $\operatorname{Re}\mathfrak{F}_{\mathbf{k}}$ 是紫外有限的。紫外发散确实出现在 $\operatorname{Im}\mathfrak{F}_{\mathbf{k}}$ 中，且可以按照常规有效场论方法重整化到有效拉格朗日量的参数中 (例如，参见文献 [34] 中的综述)。由于式 (140) 的第二行描述刘维尔演化，它不会对退相干产生贡献，因此退相干仅依赖于 $\operatorname{Re}\mathfrak{F}_{\mathbf{k}}$，结果就是紫外有限的。

In the super-Hubble regime ($k\eta \to 0$) $\operatorname{Re}\mathfrak{F}_{\mathbf{k}}$ is given by

在超哈勃区域，($k\eta \to 0$) $\operatorname{Re}\mathfrak{F}_{\mathbf{k}}$ 由下式给出

$$\operatorname{Re}\mathfrak{F}_{\mathbf{k}}(\eta,\eta_{\text{in}}) \simeq \frac{3\varepsilon_1 H^2 k^2}{1024\pi^2 M_p^2}\left\{\frac{20\pi}{(-k\eta)^2} + \frac{g(k_*,-k\eta_{\text{in}})}{(-k\eta)} + \mathcal{O}\left[(-k\eta)^0\right]\right\}, \tag{142}$$

where the overall factor of 3 arises as $2+1$ where the 2 comes from the tensor environment C^T and the 1 from the scalar environment C^S. This form is universal in the sense that all details like the precise position k_* of the system/environment split and the initial time η_{in} where system and environment start off uncorrelated appear only in subdominant terms, such as the known function $g(k_*,-k\eta_{\text{in}})$ in (142). Notice that the universal leading term grows strongly at late times.

其中整体因子 3 来自 $2+1$:2 来自张量环境 C^T，1 来自标量环境 C^S。该形式是普适的，即所有细节，比如系统/环境拆分的精确位置 k_*、系统和环境初始无关联的初始时刻 η_{in}，都只出现在次主导项中，例如式 (142) 中的已知函数 $g(k_*,-k\eta_{\text{in}})$。注意到普适的主导项在晚期会快速增长。

Equation (140) can be solved explicitly [55] and because its right-hand side is quadratic in v_{sys} it returns a Gaussian state whose time evolution can be solved in great detail. We confine ourselves to exploring one consequence of (140): its implications for the 'purity' of the observed system, defined by

式 (140) 可以显式求解 [55]，且由于其等号右侧是 v_{sys} 的二次型，解出的态是高斯态，其时间演化可以非常详细地求解。我们这里仅探讨式 (140) 的一个结论: 它对被观测系统“纯度”的影响，纯度定义为

$$\mathfrak{p}_{\mathbf{k}}(\eta) := \operatorname{Tr}\left[\rho_{\mathbf{k}}^2(\eta)\right] =: \frac{1}{\sqrt{1+\Xi_{\mathbf{k}}(\eta)}}. \tag{143}$$

Purity is measure of the state’s decoherence because it satisfies $0 \le \mathfrak{p}_{\mathbf{k}} \le 1$, with $\mathfrak{p}_{\mathbf{p}} = 1$ if and only if $\rho_{\mathbf{k}}$ is a pure state and so $\rho_{\mathbf{k}}$ is also pure if and only if $\Xi_{\mathbf{k}} = 0$. Decoherence is said to be effective when $\mathfrak{p}_{\mathbf{k}} \ll 1$. Equation (140) implies

纯度是对状态退相干程度的度量，因为它满足 $0 \le \mathfrak{p}_{\mathbf{k}} \le 1$: 当且仅当 $\rho_{\mathbf{k}}$ 是纯态时 $\mathfrak{p}_{\mathbf{p}} = 1$ 成立，因此当且仅当 $\Xi_{\mathbf{k}} = 0$ 时 $\rho_{\mathbf{k}}$ 也是纯态。当 $\mathfrak{p}_{\mathbf{k}} \ll 1$ 时退相干是有效的。式 (140) 给出

$$\Xi_{\mathbf{k}}(\eta) = 8\int_{\eta_{\text{ in}}}^{\eta} \mathrm{d}\eta' \operatorname{Re}\left[\mathfrak{F}_{\mathbf{k}}(\eta', \eta_{\text{ in}})\right] P_{vv}(k, \eta'), \tag{144}$$

where P_{vv} is the power spectrum, given for the Bunch-Davies vacuum by $|u_{\mathbf{k}}(\eta)|^2$ with mode functions as given in (102). See [55] for more details.

其中 P_{vv} 是功率谱，对邦奇-戴维斯真空，功率谱由 $|u_{\mathbf{k}}(\eta)|^2$ 给出，其模式函数如式 (102) 所示。更多细节参见文献 [55]。

The strong growth of $\mathfrak{F}_{\mathbf{k}}(\eta', \eta_{\text{ in}})$ for $k\eta' \to 0$ implies the integral is dominated by the super-Hubble limit $-k\eta \le -k\eta_{\text{ in}} \ll 1$, where the universal form seen in (142) applies. Using this leads to the late-time prediction

$\mathfrak{F}_{\mathbf{k}}(\eta', \eta_{\text{ in}})$ 对应 $k\eta' \to 0$ 的强增长意味着积分由超哈勃极限 $-k\eta \le -k\eta_{\text{ in}} \ll 1$ 主导，该极限适用式 (142) 中的普适形式。利用该性质可得到晚期预言

$$\Xi_{\mathbf{k}}(\eta) \simeq \frac{5\varepsilon_1}{64\pi^2}\left(\frac{H^2}{M_p^2}\right)\frac{1}{(-k\eta)^3} = \frac{5\varepsilon_1}{64\pi^2}\left(\frac{H^2}{M_p^2}\right)\left(\frac{aH}{k}\right)^3. \tag{145}$$

The dependence on ε_1 and H/M_p found here follows directly from the couplings appearing in the underlying interaction (134), and if linearized in $\Xi_{\mathbf{k}}$ agrees (up to normalization) with the perturbative result found in [56]. But linearization of (145) is not required because in the small $k\eta$ limit the universal form of $\mathfrak{F}_{\mathbf{k}}$ implies it is independent of $\eta_{\text{ in}}$, and this allows (140) to be used to resume late-time behaviour. So although use of perturbative methods requires ε_1 and H/M_p to be small, the solutions to the evolution equation (140) can be trusted even for times late enough that $\Xi_{\mathbf{k}}$ is not small because the factor of a^3 is large enough to compensate for the small perturbative prefactors.

本文得到的对 ε_1 和 H/M_p 的依赖关系，直接源于基础相互作用 (134) 中出现的耦合，若对 $\Xi_{\mathbf{k}}$ 做线性化处理，结果 (在归一化范围内) 与文献 [56] 得到的微扰结果一致。但我们无需对 (145) 做线性化，因为在小 $k\eta$ 极限下，$\mathfrak{F}_{\mathbf{k}}$ 的普适形式表明它与 $\eta_{\text{ in}}$ 无关，这使得我们可以用 (140) 重新总结晚期行为。因此，尽管微扰方法要求 ε_1 和 H/M_p 为小量，即使在 $\Xi_{\mathbf{k}}$ 因 a^3 因子足够大、足以抵消小的微扰前置因子而不再为小的足够晚的时间，演化方程 (140) 的解仍然可信。

Notice that because these arguments explicitly use proximity to de Sitter (by perturbing in ε_1) the prediction (145) applies at the end of inflation, and not at the much later epoch when the observed modes re-enter the Hubble scale and become observed. At this writing it is an open question how the purity evolves during the post-inflationary universe, but it is intuitive that a very classical state is not expected to be recohered by the later evolution of the universe.

注意，由于这些推导明确利用了近德西特空间的条件 (通过对 ε_1 做微扰)，因此预言式 (145) 适用于暴胀末期，而非观测模式重新进入哈勃尺度并被观测到的晚得多的时期。截至目前，纯度在暴胀后宇宙中的演化仍是一个开放问题，但根据直觉，经典演化后期不大可能让原本已经退相干的状态重新相干。

Black Holes

黑洞

We finally consider preliminary applications of Open EFT techniques to black holes, whose static properties are captured (for non-rotating black holes) for many purposes by the metric

我们最后来探讨开放有效场论 (Open EFT) 技术在黑洞中的初步应用，对于非旋转黑洞，其静态性质在很多场景下都可由下述度规描述

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\theta^2 + r^2\sin^2\theta d\varphi^2, \tag{146}$$

where $f(r) = 1 - r_s/r$ for the simplest case: a Schwarzschild black hole with Schwarzschild radius $r_s = 2GM$. This metric has the same form as in (106), with $\gamma_{ij} = \mathrm{diag}\left[f^{-1}, r^2, r^2\sin^2\theta\right]$.

其中最简单的情况即史瓦西黑洞满足 $f(r) = 1 - r_s/r$ ，其史瓦西半径为 $r_s = 2GM$ 。该度规的形式与 (106) 式相同，满足 $\gamma_{ij} = \mathrm{diag}\left[f^{-1}, r^2, r^2\sin^2\theta\right]$.

The difficulty doing explicit calculations for black hole backgrounds means much less is known about open-system behaviour for these geometries. We therefore content ourselves to briefly describing some simple examples - such as qubit evolution - for which explicit calculations can in some circumstances be done. We also explore the properties of a toy model of a black hole for which the observed system is a field, using the technique of influence functionals.

在黑洞背景下完成显式计算难度很高，因此这类几何的开放系统性质我们目前所知甚少。因此我们仅简要介绍部分简单例子——比如量子比特演化——这类例子在某些条件下可以完成显式计算。我们还利用影响泛函技术，探究了一个黑洞玩具模型的性质，该模型中被观测系统是一个场。

Qubit Thermalization

量子比特热化

The qubit set-up proceeds similar to previous sections and we pick the qubit to hover at a fixed position in space (in Schwarzschild coordinates), so

量子比特的设置与前文类似，我们让量子比特悬浮在空间中固定位置 (史瓦西坐标系下)，因此

$$y^{\mu}(\tau)=[t(\tau),r(\tau),\theta(\tau),\varphi(\tau)]=\left[\frac{\tau}{\sqrt{1-r_s/r_0}},r_0,\theta_0,\varphi_0\right], \tag{147}$$

where τ gives proper time along the curve and r_0,θ_0,φ_0 are all constants. This trajectory is not a geodesic and so the qubit shares many properties with the uniformly accelerated Rindler example considered previously. Gravitational redshift implies the energy splitting seen at infinity between the two qubit levels is

其中 τ 给出沿该曲线的固有时，r_0,θ_0,φ_0 均为常数。该轨迹不是测地线，因此该量子比特与我们之前研究的匀加速林德勒例子有许多共同性质。引力红移意味着在无穷远处观测到的两个量子比特能级之间的能级劈裂为

$$\omega_{\infty}=\omega\sqrt{1-\frac{r_s}{r_0}}, \tag{148}$$

where ω is the splitting in the qubit's rest frame at the qubit's position.

其中 ω 是量子比特位置处，量子比特静止系中的劈裂。

We take our quantum field environment again to be a massless real scalar field, with action as in (54) but with $\lambda=0$ since we ignore field self-interactions. There are a variety of 'vacuum' states in which such a field could be prepared, including the Boulware [57], Hartle-Hawking [58] or Unruh [23] vacua, and in principle qubit response requires calculating the autocorrelation function $W(x,x')=\langle\Omega|\phi(x)\phi(x')|\Omega\rangle$ for two points along the qubit trajectory using the state of interest.

我们再次将量子场环境取为无质量实标量场，其作用量和 (54) 相同，但由于我们忽略场自相互作用，因此为 $\lambda=0$。这类场可以制备在多种“真空”态中，包括 Boulware 真空 [57]、Hartle-Hawking 真空 [58] 和 Unruh 真空 [23]，原则上量子比特的响应需要针对我们感兴趣的态，计算沿量子比特轨迹两点的自相关函数 $W(x,x')=\langle\Omega|\phi(x)\phi(x')|\Omega\rangle$。

This evaluation is particularly simple if the invariant separation between the two field points is sufficiently small that the geodesic distance, $s(x,x')$, between them is much smaller than the local curvature scale, in which case it is dominated by the universal Hadamard form [59-61] that dominates the coincident limit:

如果两个场点之间的不变间隔足够小, 使得它们之间的测地线距离 $s(x,x')$ 远小于局域曲率标度, 那么计算会变得格外简单，这种情况下，重合极限由通用阿达马形式主导 [59-61]:

$$\langle\Omega|\phi(x)\phi(x')|\Omega\rangle\simeq\frac{1}{8\pi^2\sigma(x,x')+i\varepsilon[\mathcal{T}(x)-\mathcal{T}(x')]}\quad(x\to x'), \tag{149}$$

where $\sigma(x,x')=\frac{1}{2}s^2(x,x')$ is the so-called Synge world function and $\mathcal{T}$ is any future-increasing function of time which gets multiplied by the regulator ε so that the singularity structure of (149) is that of the Wightman function. This limit applies to any state of Hadamard form and reflects the intuitive fact that physical vacuum states on curved spacetimes should be indistinguishable from their flat-space counterparts so long as one probes wavelengths much shorter than the local radius of curvature. The Hartle-Hawking and Unruh vacua in particular are Hadamard states.

其中 $\sigma(x,x') = \frac{1}{2}s^2(x,x')$ 是所谓的辛奇世界函数，$\mathcal{T}$ 是任意时间未来递增函数，它乘以调节因子 ε 后，使得 (149) 的奇点结构与怀特曼函数一致。该极限适用于任意阿达马形式的态，也印证了一个直观结论: 只要探测的波长远小于局域曲率半径，弯曲时空上的物理真空态就应和平直时空对应态不可区分。Hartle-Hawking 真空和 Unruh 真空就属于阿达马态。

Evaluating (149) along the trajectory (147) for the specific case of a Schwarzschild geometry leads to the expression for $W(\tau) = \langle\Omega|\phi[y(\tau)]\,\phi[y(0)]|\Omega\rangle$:

针对史瓦西几何的具体情况，沿轨迹 (147) 计算 (149)，可得到 $W(\tau) = \langle\Omega|\phi[y(\tau)]\,\phi[y(0)]|\Omega\rangle$ 的表达式:

$$W(\tau) \simeq -\frac{1}{64\pi^2 r_s^2\left(1-\frac{r_s}{r_0}\right)\left[\sinh\left(\frac{t(\tau)}{4r_s}\right)-i\varepsilon\right]^2}, \tag{150}$$

which holds in the regime $|\sigma[y(\tau), y(0)]| \ll r_s^2$, or equivalently

该式在 $|\sigma[y(\tau), y(0)]| \ll r_s^2$ 成立，或者等价于

$$\left(1-\frac{r_s}{r_0}\right)\sinh^2\left[\frac{t(\tau)}{4r_s}\right] \ll 1. \tag{151}$$

Here $t(\tau) = \tau(1-r_s/r_0)^{-1/2}$ denotes the Schwarzschild time as measured along the qubit trajectory (147).

此处 $t(\tau) = \tau(1-r_s/r_0)^{-1/2}$ 表示沿量子比特轨迹 (147) 测量的史瓦西时间。

Comparing (150) with (78) shows that $W(\tau)$ falls off in a suggestively thermal way so one might hope to deploy the Open EFT formalism to capture late-time Markovian behaviour over timescales much longer than the fall-off time. The trick is to do so while remaining within the regime (151) for which (150) is valid. Happily $|\sigma(x,x')| \ll r_s^2$ can be consistent with late times $t(\tau) \gg r_s$ provided r_0 is chosen close enough to the horizon to ensure [62]

将 (150) 与 (78) 对比可知，$W(\tau)$ 按具有热特征的方式衰减，因此我们可以尝试利用开放 EFT 形式来描述远长于衰减时间尺度上的 late-time 马尔可夫行为。关键在于操作过程中始终要满足 (150) 成立的范围 (151)。幸运的是，只要将 r_0 选得足够靠近视界以保证 [62]，$|\sigma(x,x')| \ll r_s^2$ 就可以满足 late time $t(\tau) \gg r_s$ 的条件

$$1 \ll \frac{t(\tau)}{r_s} \ll \left|2\log\left(\frac{1-r_s/r_0}{4}\right)\right|. \tag{152}$$

Having control of the approximation (150) for $W(\tau)$ in this manner, one proceeds as for the uniformly accelerated qubit of section ”Accelerated Qubit Thermalization”. The same arguments as given above show that Markovian evolution is valid in the limit where

通过这种方式控制 $W(\tau)$ 的近似式 (150)，我们就可以按照“加速量子比特热化”一节中处理均匀加速量子比特的方法进行推导。和上文给出的论证相同，马尔可夫演化在如下极限下成立:

$$1 \gg 4\pi r_s \omega_\infty \gg \frac{g^2}{2\pi}, \tag{153}$$

and when this is true the resummed Markovian evolution at late times describes qubit thermalization to its local Hawking temperature

当该条件满足时，重整后的马尔可夫演化在晚期会描述量子比特热化至其局部霍金温度的过程

$$T_H(r_0) = \frac{(4\pi r_s)^{-1}}{\sqrt{1 - r_s/r_0}}, \tag{154}$$

for which $\beta(r_0)\,\omega = 4\pi r_s \omega \sqrt{1 - r_s/r_0} = 4\pi r_s \omega_\infty$.

此时满足 $\beta(r_0)\,\omega = 4\pi r_s \omega \sqrt{1 - r_s/r_0} = 4\pi r_s \omega_\infty$ 。

The thermalization timescales as seen by the observer at infinity are found to be

我们发现，无穷远观测者观测到的热化时标为

$$\xi_{c\infty} = 2\xi_{d\infty} = \frac{4\pi \tanh(2\pi r_s \omega_\infty)}{g^2 \omega_\infty} \simeq \frac{8\pi^2 r_s}{g^2} \tag{155}$$

which correspond to the blue-shifted timescales $\xi_{c,d}(r_0) = \xi_{c,d}\infty\sqrt{1 - r_s/r_0}$ in the qubit's frame. Because all scales redshift in the same way we have $\xi_\infty T_H = \xi(r_0)\, T_H(r_0)$ and so the same hierarchy of timescales seen by the qubit is also seen at infinity.

这对应量子比特参考系中蓝移后的时标 $\xi_{c,d}(r_0) = \xi_{c,d}\infty\sqrt{1 - r_s/r_0}$ 。由于所有尺度都以相同方式红移，我们得到 $\xi_\infty T_H = \xi(r_0)\, T_H(r_0)$ ，因此量子比特自身观测到的时标层级和无穷远观测者观测到的一致。

Hotspots and Influence Functionals

热点与影响泛函

A hindrance when studying interacting quantum fields near black holes is the relative lack of explicit calculations that include self-interactions in addition to the interaction with the classical gravitational field (see however e.g. [63, 64]). Although qubit calculations in a black hole background can be informative, they are also exceedingly simple and so might not capture features that arise when more complicated systems involving fields are observed. It is for these more complicated systems that influence functionals also come into their own.

在黑洞附近研究相互作用量子场时，一个阻碍是，除了与经典引力场的相互作用外，包含自相互作用的显式计算相对较少 (不过参见例如文献 [63, 64])。虽然黑洞背景下的量子比特计算能提供信息，但这类计算过于简单，因此可能无法涵盖观测包含场的更复杂系统时出现的特征。而正是针对这类更复杂的系统，影响泛函才能发挥自身作用。

In this section we illustrate the use of many of the tools described above in a slightly more complicated field-theoretical system. Because we do not yet have good black hole examples to hand we instead illustrate their use using a simpler solvable system, called a 'hotspot' [65-67], that captures some of the features of a localized thermal source.

在本节中，我们将举例说明上述诸多工具在一个稍复杂的场论系统中的应用。由于我们目前还没有现成的合适黑洞案例，我们改用一个更简单的可解系统“热点”[65-67] 来演示这些工具的用法，该系统可以体现局域热源的部分特征。

Hotspots: Small Hot Sources

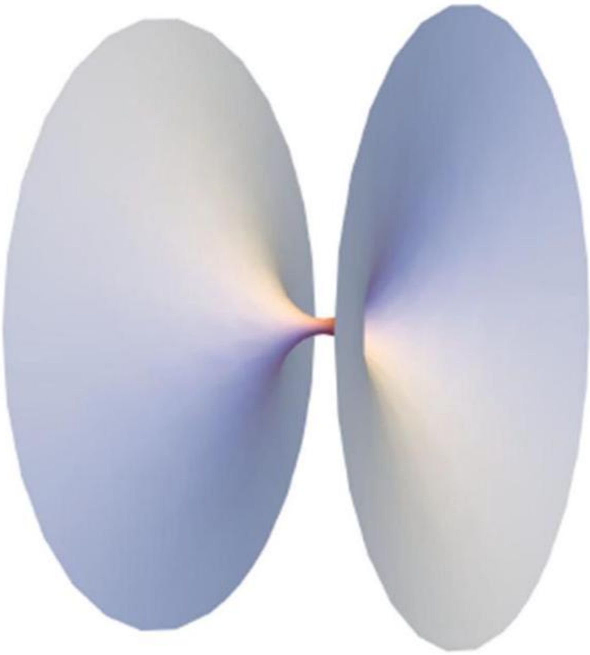
热点: 小型热源

A hotspot is a simple field-theoretic system for which environment degrees of freedom are a collection of N massless scalar fields χ^a localized to an infinite spatial region $\mathcal{R}_B$ of spacetime and prepared in a thermal state. This is meant to emulate a black hole whose interior is hidden behind an event horizon. $\mathcal{R}_B$ is taken to be infinite so as to ensure that the fields do not have gaps in the spacing of their particle states. The observable sector representing the exterior of a black hole is a single massless real scalar field ϕ living in a different spatial region $\mathcal{R}_A$ that is also taken to be infinitely large. This field can be prepared in any convenient state for the purposes of study, but we choose here the free-field vacuum state.

热点是一种简单的场论系统，其环境自由度是由 N 个无质量标量场 χ^a 构成，这些场局域在时空的无限空间区域 $\mathcal{R}_B$ 中，且制备于热态。该模型用来模拟内部藏在事件视界之后的黑洞。我们将 $\mathcal{R}_B$ 取为无限大，以保证这些场的粒子态能谱不存在间隙。代表黑洞外部的可观测部分是一个单一的无质量实标量场 ϕ，存在于另一个同样取为无限大的空间区域 $\mathcal{R}_A$ 中。研究中该场可以制备在任意方便的态，但本文选择自由场真空态。

Fig. 5 A cartoon of the two spatial branches, $\mathcal{R}_A$ and $\mathcal{R}_B$, in which the system field ϕ and the N environmental fields χ^a respectively live. Mixing between these fields occurs only in the localized throat region, which can be taken to be a small sphere of radius r_h (or effectively a point in the limit that r_h is much smaller than all other scales of interest)

图 5 两个空间分支 $\mathcal{R}_A$ 和 $\mathcal{R}_B$ 的示意图，系统场 ϕ 和 N 个环境场 χ^a 分别存在于这两个分支。场之间仅在局域的喉区发生混合，该区域可视为一个半径为 r_h 的小球 (当 r_h 远小于所有其他感兴趣的尺度时，可近似为一个点)



The two kinds of fields interact with one another only on the intersection of the regions, $\mathcal{R}_A \cap \mathcal{R}_B$, which is taken to be the surface of a sphere of radius r_h (meant as a proxy for the event horizon, see Fig. 5). The system is solvable if the interactions are limited to a Caldeira-Leggett style bilinear mixing of these two kinds of fields [68], since in this case the entire problem remains Gaussian. It is often of interest to focus on the far-field case where $r_h \to 0$ corresponding to measurements that are performed in the observable sector at distances much larger than r_h .

两类场仅在两个区域的交集 $\mathcal{R}_A \cap \mathcal{R}_B$ 发生相互作用，该交集是一个半径为 r_h 的球面 (作为事件视界的替代，参见图 5)。如果相互作用仅限于卡尔代拉-莱格特型的两类场双线性混合，该系统可解 [68]，此时整个问题始终是高斯型的。我们通常关注远场情形，即 $r_h \to 0$ 对应在可观测区域进行的测量，且测量位置距离远大于 r_h 。

The free actions for these two fields are taken to be

两类场的自由作用量取为

$$S_{A0}[\phi] := -\frac{1}{2}\int_{\mathcal{R}_A} \mathrm{d}^4x \partial_\mu\phi\partial^\mu\phi \;\text{ and }\; S_B[\chi] := -\frac{1}{2}\int_{\mathcal{R}_B} \mathrm{d}^4x \delta_{ab}\partial_\mu\chi^a\partial^\mu\chi^b.$$

(156)

Although the gravitational field of the environment can be included by assigning a Schwarzschild geometry to region $\mathcal{R}_A$ (with $r_h > 2GM$ unless we want this to actually be a black hole), we here take both $\mathcal{R}_A$ and $\mathcal{R}_B$ to be flat for simplicity. While this does not mimic the perfect infall near a black hole horizon, it does capture a localized interaction with a thermal object and so can act as a benchmark against which quantum black hole calculations can be compared.

尽管环境的引力场可以通过给区域 $\mathcal{R}_A$ 赋予史瓦西几何来纳入 (若我们要求它是真实黑洞则保留 $r_h > 2GM$，否则无需)，为简化起见本文假设 $\mathcal{R}_A$ 和 $\mathcal{R}_B$ 都是平直的。虽然这无法完全模拟黑洞视界附近的自由下落，但是它确实抓住了与热物体局域相互作用的特征，因此可以作为量子黑洞计算的对比基准。

In the limit $r_h \to 0$ the Caldeira-Leggett style interaction between the two sectors is given by a mixing term of the form

在 $r_h \to 0$ 极限下，两个部分之间的卡尔代拉-莱格特型相互作用由如下形式的混合项给出

$$S_{\text{int}} = -g_a \int \mathrm{d}t \chi^a(t, \mathbf{0}) \phi(t, \mathbf{0}) \tag{157}$$

so that the fields interact only at a single hotspot point $\mathbf{x} = \mathbf{0}$. There is an implied sum over the N couplings g_a in (157), and it is assumed that the couplings are of the same size: $g_a = \tilde{g}/\sqrt{N}$ for all a (with the factor $\sqrt{N}$ extracted for later convenience).

因此场仅在单个热点位置 $\mathbf{x} = \mathbf{0}$ 发生相互作用。(157) 中隐含了对 N 个耦合 g_a 的求和，且假设所有耦合大小相同: 对所有 a 有 $g_a = \tilde{g}/\sqrt{N}$ (为方便后续计算提取出因子 $\sqrt{N}$)。

This interaction also implies the existence of another localized self-interaction,

该相互作用还隐含存在另一个局域自相互作用，

$$S_{\text{ct}} = -\frac{\lambda}{2} \int \mathrm{d}t \phi^2(t, \mathbf{0}), \tag{158}$$

which must be present in order to absorb some of the UV divergences that (157) generates. The treatment of these divergences associated with couplings to localized sources we handle using the general formalism developed in [69]. Because S_{ct} does not couple sectors A and B it is often convenient to combine it with S_{A0} from (156) and write

它是吸收 (157) 产生的部分紫外发散所必需的。我们采用文献 [69] 中发展的一般形式来处理这类与局域源耦合相关的发散。由于 S_{ct} 不耦合 A 区和 B 区，通常可方便地将它和 (156) 中的 S_{A0} 合并，写作

$$S_A = S_{A0} + S_{\text{ct}} = -\frac{1}{2} \int_{\mathcal{R}_A} \mathrm{d}^4x \left[\partial_\mu \phi \partial^\mu \phi + \lambda \delta^3(\mathbf{x}) \phi^2(t, \mathbf{0})\right]. \tag{159}$$

The initial conditions at $t = 0$ for the full density matrix are assumed to be uncorrelated - cf. Eq. (16):

整个系统密度矩阵在 $t = 0$ 处的初始条件假设为无关联，参见式 (16):

$$\rho_0 = \rho(t = 0) = \rho_{A0} \otimes \rho_B \tag{160}$$

with ϕ prepared in its vacuum $\rho_{A0} = |\Omega\rangle\langle\Omega|$ and the environment fields χ^a prepared in a thermal state at temperature $1/\beta$:

其中 ϕ 处于真空态 $\rho_{A0} = |\Omega\rangle\langle\Omega|$，环境场 χ^a 处于温度为 $1/\beta$ 的热态:

$$\rho_B = \frac{e^{-\beta H_B}}{\mathrm{Tr}\left[e^{-\beta H_B}\right]} \quad \text{with} \quad H_B = \frac{1}{2}\int_{\mathcal{R}_B} \mathrm{d}^3 x \delta_{ab}\left[\partial_t \chi^a \partial_t \chi^b + \nabla\chi^a \cdot \nabla\chi^b\right]. \tag{161}$$

With this choice $\langle\varphi_1|\rho_{A0}|\varphi_2\rangle = \Psi[\varphi_1]\Psi^*[\varphi_2]$ where $\Psi[\varphi] := \langle\varphi \mid \Omega\rangle$ is the wave-functional of the vacuum.

按照这个选择 $\langle\varphi_1|\rho_{A0}|\varphi_2\rangle = \Psi[\varphi_1]\Psi^*[\varphi_2]$，其中 $\Psi[\varphi] := \langle\varphi \mid \Omega\rangle$ 是真空的波泛函。

In general, interactions introduce correlations between ϕ and χ^a spoiling the factorized form of (160) and for this reason we assume that the interaction S_{int} is suddenly turned on at $t = 0$ - this choice allows one to prepare the initially uncorrelated state and then observe how the combined system reacts to the onset of couplings (sometimes called a quench). One can think of this sudden approximation as a cartoon of the formation of a black hole, with outgoing transient waves radiating out from the spacetime event where the hotspot is formed.

一般而言，相互作用会在 ϕ 和 χ^a 之间引入关联，破坏 (160) 的因式分解形式，因此我们假设相互作用 S_{int} 在 $t = 0$ 处突然开启——这一选择可以让我们制备初始无关联的态，随后观测整个系统对耦合开启 (有时称为猝灭) 的反应。这种突然近似可以看作黑洞形成的简化模型: 热点形成的时空事件会向外辐射出瞬态波。

Because the model is Gaussian the 2-point correlators contain all of the information. In the $N \to \infty$ limit back-reaction of the external field ϕ onto the environment can be neglected, so the environment fields satisfy [14]

由于该模型是高斯型，两点关联函数包含了全部信息。在 $N \to \infty$ 极限下，外场 ϕ 对环境的反作用可以忽略，因此环境场满足 [14]

$$\underset{B}{\mathrm{Tr}}\left[\chi^a(t,\mathbf{x})\chi^b(0,\mathbf{0})\rho_B\right] = \delta^{ab}\frac{\coth\left[\frac{\pi}{\beta}(t+|\mathbf{x}|-i\varepsilon)\right] - \coth\left[\frac{\pi}{\beta}(t-|\mathbf{x}|-i\varepsilon)\right]}{8\pi\beta|\mathbf{x}|}. \tag{162}$$

---

[14] Time and spatial translation symmetry of the thermal bath is used here. Note also that the $i\varepsilon$ - prescription used here only works for real t .

[14] 这里我们利用了热库的时间和空间平移对称性。还需要注意，此处使用的 $i\varepsilon$ prescription 仅对实 t 成立。

Of particular importance later on is the special case

后续讨论中尤为重要的是如下特殊情况

$$\tilde{g}^2\mathcal{W}_B(t) := g_a g_b \mathrm{Tr}_B\left[\chi^a(t,\mathbf{0})\chi^b(0,\mathbf{0})\rho_B\right] = -\frac{\tilde{g}^2}{4\beta^2\sinh^2\left[\frac{\pi}{\beta}(t-i\varepsilon)\right]} \tag{163}$$

since this appears explicitly in the influence functional.

因为它会显式出现在影响泛函中。

More interesting is the response of the external field ϕ , which feels the effects of the environment even as $N\to\infty$. Because the system is Gaussian the ϕ response function can be computed exactly as a function of $\tilde{g},\lambda$ and spacetime position, with the result given explicitly in [65]. It suffices here to quote the result in the equal-time limit expanded out to order $\tilde{g}^2$, for which (after a renormalization of the self-coupling λ) the result is

更值得关注的是外场 ϕ 的响应，即使在 $N\to\infty$ 条件下外场也能感受到环境的影响。由于系统是高斯型，ϕ 响应函数可以作为 $\tilde{g},\lambda$ 和时空位置的函数精确计算，结果已在文献 [65] 中给出。此处我们只需给出展开到 $\tilde{g}^2$ 阶的等时极限结果，在对自耦合 λ 重正化后，结果为

$$\begin{aligned}\mathrm{Tr}_B\left[\phi_H(t,\mathbf{x})\phi_H(t,\mathbf{x}')\rho_A\right] \simeq{}& \frac{1}{4\pi^2\left[-(t-t'-i\varepsilon)^2+|\mathbf{x}-\mathbf{x}'|^2\right]}\\ &+\frac{\lambda}{16\pi^3}\left[\frac{\Theta(t-|\mathbf{x}|)}{|\mathbf{x}|}\frac{1}{(|\mathbf{x}|+i\varepsilon)^2-|\mathbf{x}'|^2}+\frac{\Theta(t-|\mathbf{x}'|)}{|\mathbf{x}'|}\frac{1}{(|\mathbf{x}'|-i\varepsilon)^2-|\mathbf{x}|^2}\right]\\ &-\frac{\tilde{g}^2\Theta(t-|\mathbf{x}|)\Theta(t-|\mathbf{x}'|)}{64\pi^2\beta^2|\mathbf{x}||\mathbf{x}'|\sinh^2\left[\frac{\pi}{\beta}(|\mathbf{x}|-|\mathbf{x}'|+i\varepsilon)\right]}\\ &+\frac{\tilde{g}^2}{32\pi^4}\left[\frac{\Theta(t-|\mathbf{x}|)}{\left[(|\mathbf{x}|+i\varepsilon)^2-|\mathbf{x}'|^2\right]^2}+\frac{\Theta(t-|\mathbf{x}'|)}{\left[(|\mathbf{x}'|-i\varepsilon)^2-|\mathbf{x}|^2\right]^2}\right]\\ &+\frac{\tilde{g}^2}{64\pi^4}\left[\frac{\delta(t-|\mathbf{x}|)}{|\mathbf{x}|\left[(t+i\varepsilon)^2+|\mathbf{x}'|^2\right]}+\frac{\delta(t-|\mathbf{x}'|)}{|\mathbf{x}'|\left[(t-i\varepsilon)^2+|\mathbf{x}|^2\right]}\right].\end{aligned} \tag{164}$$

The delta-function terms reveal an outgoing transient wave radiating out from the system’s shock at $\mathbf{x}=t=0$. The step functions show how the external system’s properties change after this wave passes. Translation symmetry is clearly broken while invariance under rotations about the hotspot position is preserved.

δ 函数项显示，有出射瞬态波从 $\mathbf{x}=t=0$ 处的系统激波处向外辐射。阶跃函数体现了该波经过后外系统性质的变化。平移对称性被明显破坏，而绕热点位置的转动不变性得以保留。

A connection to the previous qubit calculations can also be made by coupling a qubit to the ϕ -field alone at a nonzero distance from the hotspot. Doing so shows that this qubit thermalizes to the temperature of the hotspot in certain regimes of parameter space - see [66] for further details.

我们也可以将一个量子比特耦合到距离热点非零位置处的 ϕ 场，以此建立与之前量子比特计算的关联。这样做表明，在某些参数空间区域，该量子比特会热化到热点的温度——更多细节参见文献 [66]。

Hotspot Influence Functional

热点影响泛函

We now take the hotspot model and use it to illustrate some of the features of the influence functional formalism, and how it overlaps with the other approaches encountered above. As in Eq. (68), the components of the reduced Schrödinger-picture density matrix for the ϕ field have the path-integral representation

我们现在利用热点模型阐述影响泛函形式体系的若干性质，以及它和前文介绍的其他方法的共通之处。与式 (68) 一致，ϕ 场的约化薛定谔绘景密度矩阵的分量具有路径积分表示形式

$$\langle\varphi_2\left|\hat{\rho}_A(t)\right|\varphi_1\rangle = \sum_{\varphi_3,\varphi_4}\int_{\varphi_4}^{\varphi_2}\mathcal{D}\phi^+\int_{\varphi_3}^{\varphi_1}\mathcal{D}\phi^- e^{iS_A[\phi^+]-iS_A[\phi^-]+iS_{IF}[\phi^+,\phi^-]}\langle\varphi_4\left|\rho_{A0}\right|\varphi_3\rangle,$$

(165)

where the influence functional is given by (69), which in the present instance is

其中影响泛函由 (69) 给出，在当前情形下为

$$e^{iS_{IF}[\phi^+,\phi^-]} := \sum_{\chi,\chi_3,\chi_4}\int_{\chi_4}^{\chi}\mathcal{D}\chi^+\int_{\chi_3}^{\chi}\mathcal{D}\chi^- \tag{166}$$

$$\times e^{iS_B[\chi^+]+iS_{\text{int}}[\phi^+,\chi^+]-iS_B[\chi^-]-iS_{\text{int}}[\phi^-,\chi^-]}\langle\chi_4\left|\rho_B\right|\chi_3\rangle.$$

We emphasize that the actions in (166) are integrated from time $t = 0$ (where the initial conditions are applied) up to time t (where we evaluate the reduced density matrix on the left-hand side), and so the boundary conditions in the path integrations (for the field eigenstates) are applied at these times as well.

我们强调，(166) 中的作用量是从时间 $t = 0$ (施加初始条件的时刻) 积分到时间 t (我们在左侧计算约化密度矩阵的时刻)，因此路径积分 (针对场本征态) 的边界条件也施加在这两个时刻。

The leading influence functional contributions computed perturbatively in $\tilde{g}$ are

在 $\tilde{g}$ 下微扰计算得到的主导影响泛函贡献为

$$S_{IF}[\phi^+,\phi^-] \simeq \frac{i}{2}\left\langle S^2_{\text{int}}[\phi^+,\chi^+]\right\rangle_B + \frac{i}{2}\left\langle S^2_{\text{int}}[\phi^-,\chi^-]\right\rangle_B \tag{167}$$

$$-i\langle S_{\text{int}}[\phi^+,\chi^+]\,S_{\text{int}}[\phi^+,\chi^+]\rangle_B + \dots,$$

where we neglect terms $\mathcal{O}\left(\tilde{g}^3\right)$ and use the notation

其中我们忽略了项 $\mathcal{O}(\tilde{g}^3)$，并采用记号

$$\langle O\rangle_B := \sum_{\chi,\chi_3,\chi_4}\int_{\chi_4}^{\chi}\mathcal{D}\chi^+\int_{\chi_3}^{\chi}\mathcal{D}\chi^- O e^{iS_B[\chi^+]-iS_B[\chi^-]}\langle\chi_4|\rho_B|\chi_3\rangle \tag{168}$$

to denote averaging over environment fields. Notice that the average can be interpreted in terms of the Fig. 4 (with $\phi^\pm$ replaced by $\chi^\pm$), where the boundary condition at time t identifies $\chi^+(t,\mathbf{x}) = \chi^-(t,\mathbf{x}) = \chi(\mathbf{x})$. This means that time integration can be thought of as being from zero to t along the upper ’+’ branch, and then back again along the lower ’-’ branch.

表示对环境场求平均。注意该平均可以用图 4 解释 (将 $\phi^\pm$ 替换为 $\chi^\pm$)，时间 t 处的边界条件标识了 $\chi^+(t,\mathbf{x}) = \chi^-(t,\mathbf{x}) = \chi(\mathbf{x})$ 。这意味着时间积分可以理解为: 沿上方“+”分支从零到 t ，再沿下方“-”分支积分返回。

It remains to compute the three averages appearing in Eq. (167). Writing out the first term more explicitly using (157) one finds

接下来需要计算式 (167) 中的三个平均。利用 (157) 将第一项展开后可得

$$\left\langle S_{\text{int}}[\phi^+,\chi^+]^2\right\rangle_B = g_a g_b\int_0^t \mathrm{d}t'\int_0^t \mathrm{d}t''\phi^+(t',\mathbf{0})\,\phi^+(t'',\mathbf{0}) \tag{169}$$
$$\times\sum_{\chi,\chi_3,\chi_4}\int_{\chi_4}^{\chi}\mathcal{D}\chi^+\int_{\chi_3}^{\chi}\mathcal{D}\chi^-\chi^{+a}(t',\mathbf{0})\,\chi^{+b}(t'',\mathbf{0})\,e^{iS_B[\chi^+]-iS_B[\chi^-]}\langle\chi_4|\rho_B|\chi_3\rangle.$$

A standard path integration exercise [70] shows that the lower line of (169) is equivalent to the following environmental propagator in the state ρ_B :

标准路径积分练习 [70] 表明，(169) 的下一行等价于状态 ρ_B 下的如下环境传播子:

$$\mathrm{Tr}_B\left[\mathcal{T}+\left\{\chi^a(t',\mathbf{0})\,\chi^b(t'',\mathbf{0})\right\}\rho_B\right] = \delta^{ab}\mathcal{F}_B(t'-t''), \tag{170}$$

where $\mathcal{T}_+$ denotes time-ordering of the field operators on the upper ’+’ branch of the closed-time path depicted in Fig. 4, and $\mathcal{F}_B$ is the Feynman propagator for a single field in the state ρ_B . Note that $\mathcal{F}_B$ is related to the Wightman function $\mathcal{W}_B-$ given in Eq. (163) - in the standard way:

其中 $\mathcal{T}_+$ 表示图 4 所示闭合时间路径上方“+”分支上场算符的时间排序，$\mathcal{F}_B$ 是状态 ρ_B 下单场的费恩曼传播子。注意 $\mathcal{F}_B$ 按标准方式与式 (163) 给出的怀特曼函数 $\mathcal{W}_B-$ 相关:

$$\mathcal{F}_B(t'-t'') = \mathcal{W}_B(t'-t'')\,\Theta(t'-t'') + \mathcal{W}_B(t''-t')\,\Theta(t''-t'), \tag{171}$$

and (using $g_a g_b\delta^{ab} = \tilde{g}^2$) this allows (169) to be written

利用 $g_a g_b\delta^{ab} = \tilde{g}^2$ 可将 (169) 改写为

$$\left\langle S_{\text{int}}\left[\phi^+,\chi^+\right]^2\right\rangle_B = \tilde{g}^2\int_0^t \mathrm{d}t' \int_0^t \mathrm{d}t''\phi^+(t',\mathbf{0})\,\phi^+(t'',\mathbf{0})\,\mathcal{F}_B(t'-t'')$$

$$= 2\tilde{g}^2\int_0^t \mathrm{d}t' \int_0^{t'} \mathrm{d}t''\phi^+(t',\mathbf{0})\,\phi^+(t'',\mathbf{0})\,\mathcal{W}_B(t'-t''). \tag{172}$$

A similar exercise shows the second average in (167) is given by

类似推导可得 (167) 中第二个平均为

$$\left\langle S_{\text{int}}\left[\phi^-,\chi^-\right]^2\right\rangle_B = \tilde{g}^2\int_0^t \mathrm{d}t' \int_0^t \mathrm{d}t''\phi^-(t',\mathbf{0})\,\phi^-(t'',\mathbf{0})\,\mathcal{F}_B^*(t'-t'')$$

$$= 2\tilde{g}^2\int_0^t \mathrm{d}t' \int_0^{t'} \mathrm{d}t''\phi^-(t',\mathbf{0})\,\phi^-(t'',\mathbf{0})\,\mathcal{W}_B^*(t'-t''), \tag{173}$$

which again uses (171) together with the property $\mathcal{W}_B(-t) = \mathcal{W}_B^*(t)$ that is always valid for Hermitian fields. These imply that the path integration here anti-time orders the two fields on the lower '-' branch (as might have been expected given the interpretation of the lower branch in Fig. 4 as going backwards in time relative to the upper branch).

该式同样用到了 (171) 以及厄米场始终成立的性质 $\mathcal{W}_B(-t) = \mathcal{W}_B^*(t)$。这些性质说明此处的路径积分对下方“-”分支上的两个场做反时间排序 (结合图 4 中下方分支相对上方分支逆时间行进的解释, 这一结果符合预期)。

The third term in (167) involves a field on both the upper and lower branch,

(167) 的第三项同时涉及上下分支的场,

$$\langle S_{\text{int}}[\phi^+,\chi^+]\,S_{\text{int}}[\phi^-,\chi^-]\rangle_B = \tilde{g}^2\int_0^t \mathrm{d}t' \int_0^t \mathrm{d}t''\phi^+(t',\mathbf{0})\,\phi^-(t'',\mathbf{0})\,\mathcal{W}_B^*(t'-t'')$$

$$= \tilde{g}^2\int_0^t \mathrm{d}t' \int_0^{t'} \mathrm{d}t''\,[\phi^-(t',\mathbf{0})\,\phi^+(t'',\mathbf{0})\,\mathcal{W}_B(t'-t'')$$

$$+\phi^+(t',\mathbf{0})\,\phi^-(t'',\mathbf{0})\,\mathcal{W}_B^*(t'-t'')] \tag{174}$$

and so - recalling that $\tilde{g}^2\mathcal{W}_B^*(t'-t'') = g_a g_b \mathrm{Tr}_B\left[\chi^a(t'',\mathbf{0})\,\chi^b(t',\mathbf{0})\,\rho_B\right]$ – time-ordering is not enforced by the path integration. The integration is nevertheless ’ path-ordered’, however, inasmuch as the field on the lower branch (with time t'') is to the left of the field on the upper branch (with time t') in the correlator. The final equality arranges the integration range here to match the ones in (172) and (173).

因此需要注意: 路径积分并不要求 $\tilde{g}^2\mathcal{W}_B^*(t'-t'') = g_a g_b \mathrm{Tr}_B\left[\chi^a(t'',\mathbf{0})\,\chi^b(t',\mathbf{0})\,\rho_B\right]$ – 时间排序，但积分仍然是“路径有序”的——关联函数中，下方分支 (时间为 t'') 的场位于上方分支 (时间为 t') 的场的左侧。最后一个等式整理了积分区间，使之与 (172) 和 (173) 的积分区间匹配。

Combining everything allows the $\mathcal{O}\left(\tilde{g}^2\right)$ terms of the influence functional (167) to be explicitly written as

合并所有结果后，影响泛函 (167) 的 $\mathcal{O}\left(\tilde{g}^2\right)$ 项可以显式写为

$$S_{IF}\left[\phi^+,\phi^-\right] \simeq i\tilde{g}^2\int_0^t \mathrm{d}t'\int_0^{t'}\mathrm{d}t''\left[\phi^+\left(t',\mathbf{0}\right)\phi^+\left(t'',\mathbf{0}\right)\mathcal{W}_B\left(t'-t''\right)\right. \tag{175}$$

$$+\phi^-\left(t',\mathbf{0}\right)\phi^-\left(t'',\mathbf{0}\right)\mathcal{W}_B^*\left(t'-t''\right)$$

$$-\phi^-\left(t',\mathbf{0}\right)\phi^+\left(t'',\mathbf{0}\right)\mathcal{W}_B\left(t'-t''\right)$$

$$\left.-\phi^+\left(t',\mathbf{0}\right)\phi^-\left(t'',\mathbf{0}\right)\mathcal{W}_B^*\left(t'-t''\right)\right],$$

with $\mathcal{W}_B\left(t\right)$ given explicitly for the thermal environment by (163). Anything that can be computed using the reduced density matrix can be computed given S_{IF} through the connection (165). For instance ϕ equal-time correlation functions are straightforwardly evaluated and lead again to the result (164).

对于热环境，$\mathcal{W}_B\left(t\right)$ 由式 (163) 明确给出。通过关系式 (165)，只要已知 S_{IF}，就可以计算所有能用约化密度矩阵计算的量。例如，等时关联函数 ϕ 可以直接求值，最终仍得到结果 (164)。

In the remainder of this subsection we explore some of the other physical implications of the above influence functional.

在本小节余下部分，我们将探讨上述影响泛函的其他一些物理意义。

Hotspot Master Equation

热点主方程

Expression (165) ensures that the influence functional encodes all of the information contained in the reduced density matrix. This includes the ability to derive a master equation like (22) for its evolution (which was also truncated at second order in the interactions).

式 (165) 保证了影响泛函包含了约化密度矩阵中的所有信息，包括可以推导出描述其演化的形如 (22) 的主方程 (该主方程同样也截取到相互作用的二阶项)。

To derive the master equation from the influence functional one evaluates

为了从影响泛函推导主方程，需要计算

$$\frac{\partial\left\langle\varphi_2\left|\hat{\rho}_A\left(t\right)\right|\varphi_1\right\rangle}{\partial t} \simeq \frac{\left\langle\varphi_2\left|\hat{\rho}_A\left(t+\Delta t\right)\right|\varphi_1\right\rangle - \left\langle\varphi_2\left|\hat{\rho}_A\left(t\right)\right|\varphi_1\right\rangle}{\Delta t} \tag{176}$$

for small enough Δt , using (165) to compute the reduced density matrix elements. Expanding for $\Delta t \ll t$ yields

当 Δt 足够小时，利用式 (165) 计算约化密度矩阵元。对 $\Delta t \ll t$ 做展开后得到

$$\partial_t \langle \varphi_2 | \hat{\rho}_A(t) | \varphi_1 \rangle \simeq \sum_{\varphi_3,\varphi_4} \int_{\varphi_4}^{\varphi_2} \mathcal{D}\phi^+ \int_{\varphi_3}^{\varphi_1} \mathcal{D}\phi^- i\partial_t \{S_A[\phi^+] - S_A[\phi^-] + S_{IF}[\phi^+,\phi^-]\}$$

$$\times e^{iS_A[\phi^+] - iS_A[\phi^-] + iS_{IF}[\phi^+,\phi^-]} \langle \varphi_4 | \rho_{A0} | \varphi_3 \rangle \tag{177}$$

up to terms $\mathcal{O}(\Delta t)$. The two terms depending on $\partial_t S_A$ can be re-expressed in operator language in terms of the Hamiltonian corresponding to S_A , which given (159) is

展开至项 $\mathcal{O}(\Delta t)$ 为止。两个依赖 $\partial_t S_A$ 的项可以用算符语言，通过对应于 S_A 的哈密顿量重新表示，结合式 (159) 可得该哈密顿量为

$$H_A = \frac{1}{2} \int \mathrm{d}^3\mathbf{x} \left[(\partial_t \phi)^2 + |\nabla\phi|^2 \right] + \frac{\lambda}{2} \phi^2(t, \mathbf{0}), \tag{178}$$

giving $\partial_t \langle \varphi_2 | \hat{\rho}_A(t) | \varphi_1 \rangle \simeq -i \langle \varphi_2 | [H_A, \hat{\rho}_A] | \varphi_1 \rangle + (\partial_t S_{IF}$ term) .

得到 $\partial_t \langle \varphi_2 | \hat{\rho}_A(t) | \varphi_1 \rangle \simeq -i \langle \varphi_2 | [H_A, \hat{\rho}_A] | \varphi_1 \rangle + (\partial_t S_{IF}$ term) 。

To simplify the final term, we evaluate $\partial_t S_{IF}$ to leading nontrivial order in $\tilde{g}$, using (175) to write

为了化简最后一项，我们将 $\partial_t S_{IF}$ 计算至 $\tilde{g}$ 的最低非平庸阶，利用式 (175) 写为

$$i\partial_t S_{IF} = -\tilde{g}^2 \int_0^t \mathrm{d}s \{ [\phi^+(t,\mathbf{0})\phi^+(s,\mathbf{0}) - \phi^-(t,\mathbf{0})\phi^+(s,\mathbf{0})] \mathcal{W}_B(t-s) \tag{179}$$

$$+ [\phi^-(t,\mathbf{0})\phi^-(s,\mathbf{0}) - \phi^+(t,\mathbf{0})\phi^-(s,\mathbf{0})] \mathcal{W}_B^*(t-s) \}.$$

Using $\phi^+(t,\mathbf{0}) = \varphi_2(\mathbf{0})$ and $\phi^-(t,\mathbf{0}) = \varphi_1(\mathbf{0})$, this becomes

代入 $\phi^+(t,\mathbf{0}) = \varphi_2(\mathbf{0})$ 和 $\phi^-(t,\mathbf{0}) = \varphi_1(\mathbf{0})$ 后，上式变为

$$\partial_t \langle \varphi_2 | \hat{\rho}_A(t) | \varphi_1 \rangle \simeq -i \langle \varphi_2 | [H_A, \hat{\rho}_A] | \varphi_1 \rangle - [\varphi_2(\mathbf{0}) - \varphi_1(\mathbf{0})]$$

$$\int_0^t \mathrm{d}s \mathcal{W}_B(t-s) F^+(t,s)$$

$$- [\varphi_1(\mathbf{0}) - \varphi_2(\mathbf{0})] \int_0^t \mathrm{d}s \mathcal{W}_B^*(t-s) F^-(t,s), \tag{180}$$

which introduces the shorthand

这里引入了简写记号

$$F^\pm(t,s) := \tilde{g}^2 \sum_{\varphi_3,\varphi_4} \int_{\varphi_4}^{\varphi_2} \mathcal{D}\phi^+ \int_{\varphi_3}^{\varphi_1} \mathcal{D}\phi^- \phi^\pm(s,\mathbf{0}) \tag{181}$$

$$\times e^{iS_A[\phi^+]-iS_A[\phi^-]+iS_{IF}[\phi^+,\phi^-]}\langle\varphi_4|\rho_{A0}|\varphi_3\rangle.$$

This satisfies $F^-(t,s) = F^{+*}(t,s)$, as can be seen using the identity $S^*_{IF}[\phi^+,\phi^-] = S_{IF}[\phi^-,\phi^+]$ that follows from Eq. (166). The function $F^{\pm}(t,s)$ has a simple operator interpretation if we drop terms beyond leading order in $\tilde{g}^2$, since then

> 它满足 $F^-(t,s) = F^{+*}(t,s)$，这一点可以利用由式 (166) 推导出的恒等式 $S^*_{IF}[\phi^+,\phi^-] = S_{IF}[\phi^-,\phi^+]$ 得到。如果我们舍弃 $\tilde{g}^2$ 领头阶以外的项，函数 $F^{\pm}(t,s)$ 就有简单的算符诠释，因为此时

$$\begin{aligned} F^{\pm}(t,s) &\simeq \tilde{g}^2\sum_{\varphi_3,\varphi_4}\int_{\varphi_4}^{\varphi_2}\mathcal{D}\phi^+\int_{\varphi_3}^{\varphi_1}\mathcal{D}\phi^-\phi^{\pm}(s,\mathbf{0})\,e^{iS_A[\phi^+]-iS_A[\phi^-]}\langle\varphi_4|\rho_{A0}|\varphi_3\rangle, \\ &= \tilde{g}^2\left\langle\varphi_2\left|e^{-iH_A(t-s)}\hat{\phi}(\mathbf{0})\,e^{-iH_As}\rho_{A0}e^{+iH_At}\right|\varphi_1\right\rangle. \end{aligned} \tag{182}$$

Using this in the above expressions we obtain the evolution equation,

> 将其代入上述表达式后，我们得到演化方程:

$$\partial_t\hat{\rho}_A(t) \simeq -i\left[H_A,\hat{\rho}_A(t)\right] \tag{183}$$

$$-\tilde{g}^2\int_0^t \mathrm{d}s\left\{\mathcal{W}_B(t-s)\left[\hat{\phi}(\mathbf{0}),e^{-iH_A(t-s)}\hat{\phi}(\mathbf{0})\,e^{-iH_As}\rho_{A0}e^{+iH_At}\right]+\text{ h.c. }\right\},$$

where we peel off the eigenstates to yield an operator equation, and where 'h.c.' means Hermitian conjugate of the previous terms in the curly bracket. This is more illuminating in the interaction picture, defined using H_A as the unperturbed Hamiltonian so that $\rho_A(t) := e^{+iH_At}\hat{\rho}_A(t)\,e^{-iH_At}$, since the above becomes

> 其中我们分离出本征态得到算符方程，'h.c.' 表示花括号中前面几项的厄米共轭。在利用 H_A 作为未受扰哈密顿量定义的相互作用绘景下，此时 $\rho_A(t) := e^{+iH_At}\hat{\rho}_A(t)\,e^{-iH_At}$，结果会更加清晰，因为上述方程变为

$$\partial_t\rho_A(t) \simeq -\frac{i\lambda}{2}\left[\phi^2(t,\mathbf{0}),\rho_A(t)\right] \tag{184}$$

$$-\tilde{g}^2\int_0^t \mathrm{d}s\{\mathcal{W}_B(t-s)\left[\phi(t,\mathbf{0}),\phi(s,\mathbf{0})\,\rho_{A0}\right]+\text{ h.c. }\}.$$

This is the leading perturbative result since dropping additional powers of $\tilde{g}$ is what justifies dropping S_{IF} in going from (181) to (182). Notice that because $\rho_A(s) = \rho_{A0}+\mathcal{O}\left(\tilde{g}^2\right)$ in the interaction picture, Eq. (184) agrees with the perturbative limit of the Nakajima-Zwanzig equation given in (22), which in this example would be

> 这是领头阶微扰结果，因为舍弃 $\tilde{g}$ 的额外幂次正是从 (181) 到 (182) 推导中舍弃 S_{IF} 的依据。注意到由于相互作用绘景中的 $\rho_A(s) = \rho_{A0}+\mathcal{O}\left(\tilde{g}^2\right)$，式 (184) 与式 (22) 给出的中岛-赞吉方程的微扰极限一致，在本例子中该极限为

$$\partial_t \rho_A(t) \simeq -\frac{i\lambda}{2}\left[\phi^2(t,\mathbf{0}),\rho_A(t)\right] \tag{185}$$

$$-\tilde{g}^2\int_0^t \mathrm{d}s\{\mathcal{W}_B(t-s)\left[\phi(t,\mathbf{0}),\phi(s,\mathbf{0})\rho_A(s)\right]+\text{ h.c. }\}.$$

It is fairly common practice in the literature to justify master equations like (185) by deriving (184) in perturbation theory and then simply replacing $\rho_{A0} \to \rho_A(s)$ with the justification that these agree at lowest order in $\tilde{g}^2$. Of course this argument is actually ambiguous, as we could have equivalently taken any number of other combinations of operators that agree with ρ_{A0} as $\tilde{g} \to 0$ by the same argument, and all of these choices can disagree [15] on the predicted evolution beyond order $\tilde{g}^2$. From this point of view the derivation that passes through the Nakajima-Zwanzig equation (22) is preferable, because this derives the higher-order dependence on couplings like $\tilde{g}$ by explicitly tracing out the environment order-by-order in $\tilde{g}$.

在文献中, 通常的做法是通过微扰论推导得到 (184), 然后直接将其替换为 $\rho_{A0} \to \rho_A(s)$ 来证明 (185) 这类主方程的合理性，理由是二者在 $\tilde{g}^2$ 的最低阶上结果一致。当然这个论证实际上是不明确的，因为按照同样的逻辑，我们也可以选择任意多其他算子组合，它们同样能在 $\tilde{g} \to 0$ 阶和 ρ_{A0} 保持一致，但在超出 $\tilde{g}^2$ 阶的预测演化中，所有这些选择都可能与 [15] 结果不一致。从这个角度来看，经过中岛-赞格方程 (22) 的推导更可取，因为该推导通过在 $\tilde{g}$ 中逐阶对环境显式求迹，得到了对 $\tilde{g}$ 这类耦合的高阶依赖。

For the hotspot these arguments allow a test of the validity of the Markovian limit of the Nakajima-Zwanzig equation. To this end one again notices that the correlator function $\mathcal{W}_B(t-s)$ given in (163) is sharply peaked in time for an interval set by the inverse temperature β . This allows the remainder of the integrand to be Taylor expanded about $s=t$ if the remainder of the integrand varies over timescales much longer than β . When this is true the leading contribution behaves as if $\mathcal{W}_B(t-s)$ were proportional to $\delta(t-s)$ and the upper integration limit can be taken to infinity, resulting in the approximate Markovian evolution

对于热点方法，这些论证可以检验中岛-赞格方程马尔可夫极限的有效性。为此我们可以再次发现，(163) 给出的关联函数 $\mathcal{W}_B(t-s)$ 在逆温度 β 设定的时间区间内具有显著时间峰。如果被积函数的其余部分在远长于 β 的时间尺度上变化，就可以将其在 $s=t$ 附近做泰勒展开。当该条件成立时，领头贡献就满足 $\mathcal{W}_B(t-s)$ 正比于 $\delta(t-s)$ ，且积分上限可以取到无穷，最终得到近似的马尔可夫演化:

$$\partial_t \rho_A(t) \simeq -\frac{i\lambda}{2}\left[\phi^2(t,\mathbf{0}),\rho_A(t)\right] \tag{186}$$

$$-\tilde{g}^2\int_0^\infty \mathrm{d}s\{\mathcal{W}_B(s)\left[\phi(t,\mathbf{0}),\phi(t,\mathbf{0})\rho_A(t)\right]+\text{ h.c. }\}.$$

---

[15] An example instead replaces $\rho_{A0} \simeq \rho_A(t)$ and so predicts an evolution equation - the Redfield equation - that is time-local (as opposed to involving convolutions with $\rho_A(s)$ as in (185)).

[15] 另一个例子会替换掉 $\rho_{A0} \simeq \rho_A(t)$ ，由此得到一个时间局域的演化方程——雷德菲尔德方程——这和 (185) 中涉及 $\rho_A(s)$ 卷积的形式不同。

In the present instance a sufficient condition for ensuring both $\phi(t, \mathbf{0})$ and $\rho_A(t)$ vary slowly compared to β is when the external field theory's UV cutoff Λ obeys $\beta\Lambda \ll 1$. This makes the hotspot temperature a UV scale (which for black holes also would require the event horizon size r_h to be negligibly small, as assumed above). In this limit integrating the Markovian master equation (186) to compute the correlation functions for the system gives (after a tedious computation)

在当前情况中，保证 $\phi(t, \mathbf{0})$ 和 $\rho_A(t)$ 都相比 β 缓慢变化的充分条件是外场论的紫外截断 Λ 满足 $\beta\Lambda \ll 1$ 。这使得热点温度成为一个紫外标度 (对于黑洞而言，这还要求事件视界半径 r_h 可以忽略不计，正如上文假设)。在该极限下，对马尔可夫主方程 (186) 积分计算系统关联函数后 (经过冗长计算) 得到:

$$\begin{aligned}\mathrm{Tr}_B\left[\phi_H(t, \mathbf{x})\,\phi_H(t, \mathbf{x}')\,\rho_A\right] \simeq{}& \frac{1}{4\pi^2|\mathbf{x}-\mathbf{x}'|^2} - \frac{\lambda}{16\pi^3\,|\mathbf{x}|\,|\mathbf{x}'|\,(|\mathbf{x}|+|\mathbf{x}'|)} \\ &+ \frac{\tilde{g}^2}{32\pi^3\beta\,|\mathbf{x}|\,|\mathbf{x}'|}\,\delta(|\mathbf{x}|-|\mathbf{x}'|) + \frac{\tilde{g}^2}{16\pi^4\left(|\mathbf{x}|^2-|\mathbf{x}'|^2\right)^2}\end{aligned} \tag{187}$$

which specializes to $t > |\mathbf{x}|, |\mathbf{x}'|$ (after transients of the sudden approximation have passed). This is precisely the $|\mathbf{x}|, |\mathbf{x}'| \ll \beta$ limit of the correlator given in (164) - see [67] for more details.

其特殊形式为 $t > |\mathbf{x}|, |\mathbf{x}'|$ (骤变近似的暂态过程消退之后)。这正是 (164) 给出的关联函数的 $|\mathbf{x}|, |\mathbf{x}'| \ll \beta$ 极限——更多细节见文献 [67]。

Langevin Equations and Stochastic Evolution

朗之万方程与随机演化

Another use for the path-integral influence functional formulation is to derive a stochastic evolution equation for the fields in which the environment is reduced to stochastic noise variable appearing in the equations of motion for the observed field. This type of formulation is useful when computing the time evolution of correlation functions.

路径积分影响泛函表述的另一应用是推导场的随机演化方程，在该表述中，环境被约化为观测场运动方程中出现的随机噪声变量。这种表述在计算关联函数的时间演化时十分有用。

To see how, relabel the two fields $\phi^{\pm}$ using

推导过程如下，我们对两个场 $\phi^{\pm}$ 重新标记为

$$\phi^{\mathrm{cl}} := \phi^{+} + \phi^{-} \quad \text{and} \quad \phi^{\mathrm{q}} := \phi^{+} - \phi^{-}, \tag{188}$$

The notation 'cl' stands for classical and 'q' stands for quantum, with the logic that ϕ^{cl} emerges as a macroscopic mean field in a regime where ϕ^{q} averages to zero. Fluctuations of ϕ^{q} about zero will be the source of the noise mentioned above with which ϕ^{cl} interacts.

记号“cl”代表经典，“q”代表量子，其逻辑是: ϕ^{cl} 作为宏观平均场出现在 ϕ^{q} 平均为零的区域，ϕ^{q} 相对于零的涨落就是上述噪声的来源，ϕ^{cl} 会与该噪声相互作用。

In terms of these variables the unperturbed actions become

用这些变量表示，未受扰作用量可写为

$$S_A[\phi^+] - S_A[\phi^-] = -\frac{1}{2}\int \mathrm{d}^4x \left[\partial_\mu\phi^{\text{q}}\partial^\mu\phi^{\text{cl}} + \lambda\delta^3(\mathbf{x})\,\phi^{\text{q}}\phi^{\text{cl}}\right], \tag{189}$$

while Eq. (175) for S_{IF} is

而对应 S_{IF} 的式 (175) 为

$$\begin{aligned} S_{IF}[\phi^+,\phi^-] \simeq{}& \frac{i\tilde{g}^2}{2}\int_0^t \mathrm{d}t' \int_0^t \mathrm{d}t''\,\phi^{\text{q}}(t',\mathbf{0})\,\mathrm{Re}\left[\mathcal{W}_B(t'-t'')\right]\phi^{\text{q}}(t'',\mathbf{0}) \\ & -i\tilde{g}^2\int_0^t \mathrm{d}t' \int_0^{t'} \mathrm{d}t''\,\phi^{\text{q}}(t',\mathbf{0})\,\mathrm{Im}\left[\mathcal{W}_B(t'-t'')\right]\phi^{\text{cl}}(t'',\mathbf{0}) \,. \end{aligned} \tag{190}$$

Notice that the integration limits of the first line of (190) are not time-ordered while those of the second line are.

注意 (190) 第一行的积分限不是时序的，而第二行的积分限是时序的。

The only contribution within $S_A[\phi^+] - S_A[\phi^-] + S_{IF}[\phi^+,\phi^-]$ that is quadratic in ϕ^{q} comes from the first line of (190). Furthermore the factor of i ensures that $e^{iS_{IF}}$ becomes a real Gaussian exponential in ϕ^{q} , as would have appeared for a statistical average rather than a quantum one. This makes it suggestive to use the Hubbard-Stratonovich identity [18]

在 $S_A[\phi^+] - S_A[\phi^-] + S_{IF}[\phi^+,\phi^-]$ 中，唯一对 ϕ^{q} 二次项有贡献的部分来自 (190) 的第一行。此外，因子 i 保证了 $e^{iS_{IF}}$ 成为 ϕ^{q} 上的实高斯指数，这和统计平均而非量子平均中出现的形式一致。这提示我们使用哈伯德-斯特拉托诺维奇恒等式 [18]

$$e^{-\frac{1}{2}\int \mathrm{d}t' \int \mathrm{d}t''\,\phi^{\text{q}}(t')N(t',t'')\phi^{\text{q}}(t'')} = \int \mathcal{D}v P[v]\, e^{i\int \mathrm{d}t'\,v(t')\phi^{\text{q}}(t')}, \tag{191}$$

which expresses the left-hand side in terms of a Gaussian integral over a stochastic dummy field v which is defined to have zero mean and correlation functions given by the kernel in the exponent on the left-hand side:

该恒等式将左侧表达式改写为随机哑场 v 上的高斯积分，这个随机哑场被定义为零均值，且其关联函数由左侧指数中的核给出:

$$\langle v(t)\rangle_P = 0 \;\text{ and }\; \langle v(t)\,v(t')\rangle_P = N(t,t')\,, \tag{192}$$

where the averages here denote

此处的平均表示

$$\langle O\rangle_P \ := \int \mathcal{D}vP\,[v]\,O. \tag{193}$$

Armed with this formula the influence functional (190) appearing in the path integral can be rewritten in terms of this stochastic noise field as

借助这个公式，路径积分中出现的影响泛函 (190) 可以用这个随机噪声场重写为

$$e^{iS_{IF}} = \left\langle \exp\left(i\int_0^t \mathrm{d}t'\psi\,(t')\,\phi^{\mathrm{q}}\,(t',\mathbf{0})\right)\right\rangle_P \tag{194}$$

with the definitions

其中定义为

$$\psi\,(t)\ :=\ v\,(t) - \tilde{g}^2\int_0^t \mathrm{d}t'\ \mathrm{Im}\,[\mathcal{W}_B\,(t-t')]\,\phi^{\mathrm{cl}}\,(t',\mathbf{0})\,, \tag{195}$$

and in the present case the noise kernel is

在当前情形下，噪声核为

$$N\,(t,t') = \tilde{g}^2\,\mathrm{Re}\,\mathcal{W}_B\,(t,t')\,. \tag{196}$$

The point of this exercise is that ϕ^{q} now appears only linearly in the argument $S_{\mathrm{eff}}\,[\phi^+,\phi^-]\ :=\ S_A\,[\phi^+] - S_A\,[\phi^-] + S_{IF}\,[\phi^+,\phi^-]$ of the exponential appearing in the path integral. Its integration therefore gives a functional delta function that can be used to perform the ϕ^{cl} path integral, leading to the constraint

该推导的核心在于: 路径积分的指数项的自变量 $S_{\mathrm{eff}}\,[\phi^+,\phi^-]\ :=\ S_A\,[\phi^+] - S_A\,[\phi^-] + S_{IF}\,[\phi^+,\phi^-]$ 中, ϕ^{q} 现在仅以线性形式出现。因此对 ϕ^{q} 积分后会得到一个泛函 δ 函数，可以利用它完成 ϕ^{cl} 路径积分，最终得到约束条件

$$0 = \frac{\delta S_{\mathrm{eff}}}{\delta\phi^{\mathrm{q}}} = \Box\phi\,(x) \tag{197}$$

$$+\delta^3\,(\mathbf{x})\left[-\lambda\phi\,(t,\mathbf{0}) - v\,(t) + \tilde{g}^2\int_0^t \mathrm{d}s\,\mathrm{Im}\,[\mathcal{W}_B\,(t-s)]\,\phi\,(s,\mathbf{0})\right],$$

where we rescale $\phi\ :=\ \frac{1}{2}\phi^{\mathrm{cl}}$.

此处我们对 $\phi\ :=\ \frac{1}{2}\phi^{\mathrm{cl}}$ 做了重标度。

The upshot is that correlation functions of $\phi\ =\ \frac{1}{2}\phi^{\mathrm{cl}}$ are to be computed as a stochastic average over $v\,(t)$ with correlator $N\,(t,t')$ given in terms of $\mathrm{Re}\,\mathcal{W}_B\,(t,t')$ by (196) and with the spacetime dependence of ϕ determined by the Langevin equation

最终结论是: $\phi = \frac{1}{2}\phi^{\rm cl}$ 的关联函数需要通过对 $v(t)$ 做随机平均计算，其关联子 $N(t,t')$ 由式 (196) 通过 $\mathrm{Re}\,\mathcal{W}_B(t,t')$ 给出，而 ϕ 的时空依赖由朗之万方程确定

$$\Box\phi(x) + \delta^3(\mathbf{x})\left[-\lambda\phi(t,\mathbf{0}) + \tilde{g}^2\int_0^t \mathrm{d}s\,\mathrm{Im}\left[\mathcal{W}_B(t-s)\right]\phi(s,\mathbf{0})\right] = \delta^3(\mathbf{x})\,v(t)\,,$$

(198)

with the stochastic field v appearing as a source. Notice in the particular example of the hotspot we have $\mathrm{Im}\,\mathcal{W}_B(t-s) = \delta'(t-s)/4\pi$ and so the equation is really local in time,

其中随机场 v 以源的形式出现。注意在热点的特例中我们得到 $\mathrm{Im}\,\mathcal{W}_B(t-s) = \delta'(t-s)/4\pi$ ，因此该方程实际上是时间局域的,

$$\Box\phi(x) + \delta^3(\mathbf{x})\left[\left(-\lambda + \frac{\tilde{g}^2}{4\pi}\delta(0)\right)\phi(t,\mathbf{0}) + \tilde{g}^2\partial_t\phi(t,\mathbf{0})\right] = \delta^3(\mathbf{x})\,v(t)\,. \tag{199}$$

The $\delta(0)$ divergence appearing in (199) can be absorbed into a renormalization of λ . The $\mathrm{Im}\left[\mathcal{W}_B(t-t')\right]$ term is called the dissipation kernel because it ends up introducing single time derivatives into the equation of motion for ϕ - what turns out to be a generic feature of influence functionals. It is the noise kernel $\mathrm{Re}\,\mathcal{W}_B(t-t')$ that is responsible for the decoherence seen in earlier sections.

(199) 式中出现的 $\delta(0)$ 发散可以被吸收到 λ 的重整化中。$\mathrm{Im}\left[\mathcal{W}_B(t-t')\right]$ 项被称为耗散核，因为它最终会在 ϕ 的运动方程中引入一阶时间导数——这其实是影响泛函的普遍特征。前文各节中观测到的退相干，正是由噪声核 $\mathrm{Re}\,\mathcal{W}_B(t-t')$ 导致的。

As a final note, the Langevin equation encountered here provides the same information as does a Fokker-Planck equation - like Eq. (123) encountered in section ”Stochastic Inflation” - for the evolution of the probability distribution $P[\phi]$ that ϕ inherits from the distribution $P[v]$. Doing so brings us full circle and back to the density matrix since $P[\phi] = \langle\phi|\rho_A|\phi\rangle$. For more details we refer the reader to the literature [19, 42, 71] .

最后需要说明: 本文推导的朗之万方程, 和福克-普朗克方程 (例如“随机暴胀”一节中提到的式 (123)) 包含的信息一致，二者都描述概率分布 $P[\phi]$ 的演化，而 $P[\phi]$ 是 ϕ 从分布 $P[v]$ 继承得到的。这一推导让我们绕回原点、回到密度矩阵的讨论，因为 $P[\phi] = \langle\phi|\rho_A|\phi\rangle$ 。更多细节读者可参考文献 [19, 42, 71] 。

Summary

概要

Our focus in this chapter is on open quantum systems, which we argue carry important lessons for quantum studies in gravitational fields. We focus in particular on situations where hierarchies of scale simplify calculations for these systems - what can be called Open EFTs - as is appropriate in this section devoted to effective field theories. Such simplifications are widely used throughout physics, but their use for gravitating open quantum systems still remains young.

本章我们聚焦开放量子系统，我们认为这类系统为引力场中的量子研究提供了重要启示。我们尤其关注尺度层级简化这类系统计算的情形——可称之为开放有效场论 (Open EFTs)，这契合本节专注有效场论的定位。这类简化方法已在物理学中得到广泛应用，但将其应用于引力开放量子系统的研究仍处于起步阶段。

The hope is that the gravity community can profit from the decades of study of quantum fields interacting with open systems, particularly for problems involving event horizons. The potential benefits are many and include an improved ability to understand evolution at the very late times that are both of great interest for problems like information loss, and are places where simpler traditional perturbative methods are known always to fail. Differences between Open EFTs and traditional Wilsonian ones - such as the appearance of nonlocality in time for open systems - might yet bring surprises.

我们希望引力学界能从数十年来量子场与开放系统相互作用的研究中获益，尤其是在涉及事件视界的问题上。潜在收益颇多，包括能更好地理解极晚期演化: 极晚期演化对信息损失这类问题至关重要，且传统的简单微扰方法在这类场景下向来失效。开放有效场论与传统威尔逊有效场论的差异——比如开放系统会出现时间非定域性——仍可能带来意外新发现。

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